

## C9L4 Notes

### Arithmetic and Geometric Series; Application Problems

Sum of an arithmetic series:  $s_n = \frac{n}{2}(a_1 + a_n)$

Sum of a geometric series:  $s_n = \frac{a_1(1-r^n)}{1-r}$

Evaluate each arithmetic series described.

1.  $1 + 11 + 21 + 31 \dots, n = 14$

$a_1$

$$a_{14} = 1 + (14-1)(10)$$

$$a_{14} = 131$$

$$s_{14} = \frac{14}{2}(1+131)$$

$$s_{14} = 924$$

2.  $32 + 25 + 18 + 11 \dots, n = 17$

$a_1$

$$a_{17} = 32 + (17-1)(-7)$$

$$a_{17} = -80$$

$$s_{17} = \frac{17}{2}(32-80)$$

$$s_{17} = -408$$

Determine the number of terms  $n$  in each arithmetic series.

3.  $a_1 = 17, a_n = 143, S_n = 1200$

$$s_n = \frac{n}{2}(a_1 + a_n)$$

$$1200 = \frac{n}{2}(17+143)$$

$$1200 = \frac{n}{2}(160)$$

$$1200 = 80n$$

$$n = 15 \text{ TERMS}$$

4.  $a_1 = -11, a_n = -89, S_n = -700$

$$-700 = \frac{n}{2}(-11-89)$$

$$-700 = \frac{n}{2}(-100)$$

$$-700 = -50n$$

$$n = 14 \text{ TERMS}$$

Evaluate each geometric series described.

5.  $2 + 8 + 32 + 128 \dots, n = 6$

$$S_6 = \frac{2(1-4^6)}{1-3}$$

$$S_6 = \frac{2(1-4096)}{-3}$$

$$S_6 = \frac{2(-4095)}{-3}$$

$$S_6 = \frac{-8190}{-3}$$

$$S_6 = 2730$$

6.  $-4 + 24 - 144 + 864 \dots, n = 7$

$$S_7 = \frac{-4(1-(-6)^7)}{1-(-6)}$$

$$S_7 = \frac{-4(1-(-279936))}{7}$$

$$S_7 = \frac{-4(279937)}{7}$$

$$S_7 = -159964$$

Determine the number of terms  $n$  in each geometric series.

7.  $a_1 = -1, r = 3, S_n = -1093$

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

$$-1093 = \frac{-1(1-3^n)}{1-3}$$

$$-1093 = \frac{-1(1-3^n)}{-2}$$

$$2186 = -1(1-3^n)$$

$$2186 = -1 + 3^n$$

$$2187 = 3^n$$

$$\text{LOG}_3 2187 = n$$

$$n = 7$$

8.  $a_1 = -4, r = 6, S_n = -223948$

$$-223948 = \frac{-4(1-6^n)}{1-6}$$

$$-223948 = \frac{-4(1-6^n)}{-5}$$

$$1119740 = -4(1-6^n)$$

$$-279935 = 1-6^n$$

$$-279936 = -6^n$$

$$279936 = 6^n$$

$$\text{LOG}_6 279936 = n$$

$$n = 7$$

ARITHMETIC

9. What is the 39<sup>th</sup> term in the sequence? 7, 1, -5, -11, ...

$$a_{39} = 7 + (39-1)(-6)$$

$$a_{39} = -221$$

GEOMETRIC

10. What is the sum of the first 7 terms of the series? 3 + 12 + 48 + ...

$$S_7 = \frac{3(1-4^7)}{1-4}$$

$$S_7 = \frac{3(1-4^7)}{-3}$$

$$S_7 = \frac{3(1-16384)}{-3}$$

$$S_7 = \frac{3(-16383)}{-3}$$

$$S_7 = 16383$$

11. What is the value of x in the geometric sequence?  $x, \frac{-1}{6}, \frac{1}{216}, \frac{-1}{7776}, \dots$

DIVIDE  $\frac{-1}{6}$  BY  $\frac{-1}{36}$

$$(-1/6) / (-1/36)$$

$$x = 6$$

$$(1/216) / (-1/6) = \frac{-1}{36}$$

$$(-1/7776) / (1/216) = \frac{-1}{36}$$

12. A child puts  $\$5.00$  into a piggy bank. One week later, he puts  $\$5.25$  in the bank. Two weeks later, he puts  $\$5.50$  in the bank, and so on. How much money does he put in the bank on the 29<sup>th</sup> week?  $d = \$0.25$

$$a_n = a_1 + (n-1)(d)$$

$$a_{29} = 5.00 + (29-1)(0.25)$$

$$a_{29} = \$12.00$$

13. How many terms are there in a geometric series if the first term is 4, the common ratio is 3, and the sum of the series is 354292?

$$354292 = \frac{4(1-3^n)}{1-3}$$

$$354292 = \frac{4(1-3^n)}{-2}$$

$$-708584 = 4(1-3^n)$$

$$-177146 = 1-3^n$$

$$-177147 = -3^n$$

$$177147 = 3^n$$

$$\log_3 177147 = n$$

$$n = 11 \text{ TERMS}$$

14. In an arithmetic sequence beginning with 456 and ending with 3471, how many integers are divisible by 3?

$$3471 = 456 + (n-1)(3)$$

$$3471 = 456 + 3n - 3$$

$$3471 = 453 + 3n$$

$$3018 = 3n$$

$$n = 1006 \text{ TERMS}$$