

C8L2 Notes

Asymptotes, Domain, and Intercepts of Rational Functions

VERTICAL AND HORIZONTAL ASYMPTOTES

$$f(x) = \frac{n(x)}{d(x)}$$

- a. Vertical asymptotes at real zeros of $d(x)$
- b. Horizontal asymptotes
 - a. degree of $n(x) <$ degree of $d(x)$: $y = 0$
 - b. degree of $n(x) =$ degree of $d(x)$: $y = \frac{\text{leading coefficient of } n(x)}{\text{leading coefficient of } d(x)}$
 - c. degree of $n(x) >$ degree of $d(x)$: no horizontal asymptote

Find the following for each rational function.

- a. vertical asymptote(s)
- b. domain
- c. horizontal asymptote
- d. y-intercept
- e. x-intercept(s)

1. $y = \frac{2}{x}$

- a) VA: $x = 0$
- b) D: $(-\infty, 0) \cup (0, \infty)$
- c) HA: $y = 0$
- d) Y-INT: NONE
 $y = \frac{2}{0}$
UNDEFINED
- e) X-INT: NONE
 $0 = \frac{2}{x}$
 $0 = 2$

$$2. y = \frac{3}{x+4}$$

a) VA: $x = -4$
 $x + 4 = 0$
 $x = -4$

b) D: $(-\infty, -4) \cup (-4, \infty)$

c) HA: $y = 0$

d) Y-INT: $(0, \frac{3}{4})$

$$y = \frac{3}{0+4}$$

$$y = \frac{3}{4}$$

e) X-INT: NONE

$$0 = \frac{3}{x+4}$$

$$0 \neq 3$$

$$3. f(x) = \frac{2}{x-3} + 1$$

a) VA: $x = 3$
 $x - 3 = 0$
 $x = 3$

b) $(-\infty, 3) \cup (3, \infty)$

c) HA: $y = 1$

d) Y-INT: $(0, \frac{1}{3})$

$$f(0) = \frac{2}{0-3} + 1$$

$$f(0) = -\frac{2}{3} + 1$$

$$f(0) = \frac{1}{3}$$

e) X-INT: $(1, 0)$

$$0 = \frac{2}{x-3} + 1$$

$$-1 = \frac{2}{x-3}$$

$$-1(x-3) = 2$$

$$-1x + 3 = 2$$

$$-1x = -1$$

$$x = 1$$

4. $y = \frac{x-3}{x+1}$

a) VA: $x = -1$

$$x+1=0$$

$$x = -1$$

b) D: $(-\infty, -1) \cup (-1, \infty)$

c) HA: $y = 1$

d) Y-INT: $(0, -3)$

$$y = \frac{0-3}{0+1}$$

$$y = -3$$

e) X-INT: $(3, 0)$

$$0 = \frac{x-3}{x+1}$$

$$0 = x-3$$

$$3 = x$$

5. $f(x) = \frac{x}{x^2-x-6}$

a) VA: $x = 3, -2$

$$x^2 - x - 6$$

$$(x-3)(x+2)$$

$$x-3=0 \quad x+2=0$$

$$x = 3, -2$$

b) D: $(-\infty, -2) \cup (-2, 3) \cup (3, \infty)$

c) HA: $y = 0$

d) Y-INT: $(0, 0)$

$$f(0) = \frac{0}{0^2-0-6}$$

$$f(0) = 0$$

e) X-INT: $(0, 0)$

$$0 = \frac{x}{x^2-x-6}$$

$$0 = x$$