C6L7 Notes

Inverse Relations

To find the inverse of a relation, switch x and y. Then solve for y.

State the inverse of each equation.

1. $y = 4x - 7$

$x = 4y - 7$

$+7 \quad +7$

$x + 7 = 4y$

$\frac{x+7}{4} = y$

$f^{-1}(x) = \frac{x+7}{4}$

2. $y = -3x + 5$

$x = -3y + 5$

$-5 \quad -5$

$x - 5 = -3y$

$\frac{x-5}{-3} = y$

$f^{-1}(x) = \frac{x-5}{-3}$ or $f^{-1}(x) = \frac{-1}{3}x + \frac{5}{3}$

3. $f(x) = \frac{-3}{7}x + \frac{10}{7}$

$y = \frac{-3}{7}x + \frac{10}{7}$

$x = \frac{-3}{7}y + \frac{10}{7}$ ← MULT ALL TERMS BY 7.

$7x = -3y + 10$

$-10 \quad -10$

$7x - 10 = -3y$

$\frac{7x-10}{-3} = y$

$f^{-1}(x) = \frac{7x-10}{-3}$ or $f^{-1}(x) = \frac{-7}{3}x + \frac{10}{3}$

4. $y = 3x^2 - 7$

$x = 3y^2 - 7$

$+7 \quad +7$

$x + 7 = 3y^2$

$\frac{x+7}{3} = y^2$

$\pm\sqrt{\frac{x+7}{3}} = y$

$f^{-1}(x) = \pm\sqrt{\frac{x+7}{3}}$ or

$f^{-1}(x) = \pm\left(\frac{x+7}{3}\right)^{\frac{1}{2}}$

5. $y = -8x^2 + 1$

$x = -8y^2 + 1$

$-1 \quad -1$

$x - 1 = -8y^2$

$\frac{x-1}{-8} = y^2$

$\pm\sqrt{\frac{x-1}{-8}} = y$

$f^{-1}(x) = \pm\sqrt{\frac{x-1}{-8}}$ or $f^{-1}(x) = \pm\sqrt{\frac{-1}{8}x + \frac{1}{8}}$

or $f^{-1}(x) = \pm\left(\frac{x-1}{8}\right)^{\frac{1}{2}}$

6. $f(x) = 5 - x^3$

$y = 5 - x^3$

$x = 5 - y^3$

$-5 \quad -5$

$x - 5 = -y^3$

$-x + 5 = y^3$

$\sqrt[3]{-x+5} = \sqrt[3]{y^3}$

$\sqrt[3]{-x+5} = y$

$f^{-1}(x) = \sqrt[3]{-x+5}$ or $f^{-1}(x) = (-x+5)^{\frac{1}{3}}$

7. $f(x) = \sqrt[5]{x-2}$

$y = (x-2)^{\frac{1}{5}}$

$(x)^5 = \left((y-2)^{\frac{1}{5}}\right)^5$

$x^5 = y - 2$

$+2 \quad +2$

$x^5 + 2 = y$

$f^{-1}(x) = x^5 + 2$

8. $f(x) = \sqrt[3]{x+5} - 1$

$y = (x+5)^{\frac{1}{3}} - 1$

$x = (y+5)^{\frac{1}{3}} - 1$

$x+1 = (y+5)^{\frac{1}{3}}$

$(x+1)^3 = y + 5$

$(x+1)^3 - 5 = y$

$f^{-1}(x) = (x+1)^3 - 5$

9. $y = (x-3)^7$

$x = (y-3)^7$

$x^{\frac{1}{7}} = y - 3$

$x^{\frac{1}{7}} + 3 = y$

$f^{-1}(x) = x^{\frac{1}{7}} + 3$

OR

$f^{-1}(x) = \sqrt[7]{x} + 3$

10. $f(x) = (x+2)^3 + 4$

$y = (x+2)^3 + 4$

$x = (y+2)^3 + 4$

$x - 4 = (y+2)^3$

$(x-4)^{\frac{1}{3}} = y + 2$

$(x-4)^{\frac{1}{3}} - 2 = y$

$f^{-1}(x) = (x-4)^{\frac{1}{3}} - 2$ OR $f^{-1}(x) = \sqrt[3]{x-4} - 2$

11. $y = \frac{1}{2}(x+5)^3 - 7$

$x = \frac{1}{2}(y+5)^3 - 7$

$x + 7 = \frac{1}{2}(y+5)^3$

$2x + 14 = (y+5)^3$

$(2x+14)^{\frac{1}{3}} = y + 5$

$(2x+14)^{\frac{1}{3}} - 5 = y$

$f^{-1}(x) = (2x+14)^{\frac{1}{3}} - 5$

OR

$f^{-1}(x) = \sqrt[3]{2x+14} - 5$

12. $f(x) = \frac{1}{3}(x-1)^5 - 4$

$y = \frac{1}{3}(x-1)^5 - 4$

$x = \frac{1}{3}(y-1)^5 - 4$

$x + 4 = \frac{1}{3}(y-1)^5$

$3x + 12 = (y-1)^5$

$(3x+12)^{\frac{1}{5}} = y - 1$

$(3x+12)^{\frac{1}{5}} + 1 = y$

$f^{-1}(x) = (3x+12)^{\frac{1}{5}} + 1$

OR

$f^{-1}(x) = \sqrt[5]{3x+12} + 1$

Graph the equation of the line. State the equation of its inverse, and graph the inverse.

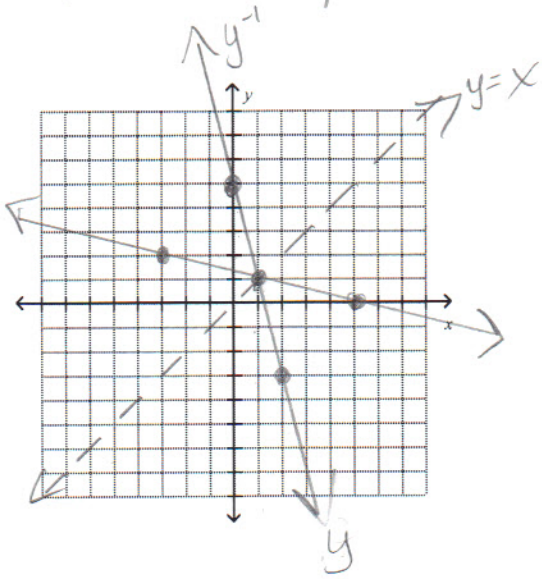
13. $y = -4x + 5$

$x = -4y + 5$

$x - 5 = -4y$

$\frac{x-5}{-4} = y$

$f^{-1}(x) = \frac{x-5}{-4}$



14. $y = \frac{2}{5}x - 7$

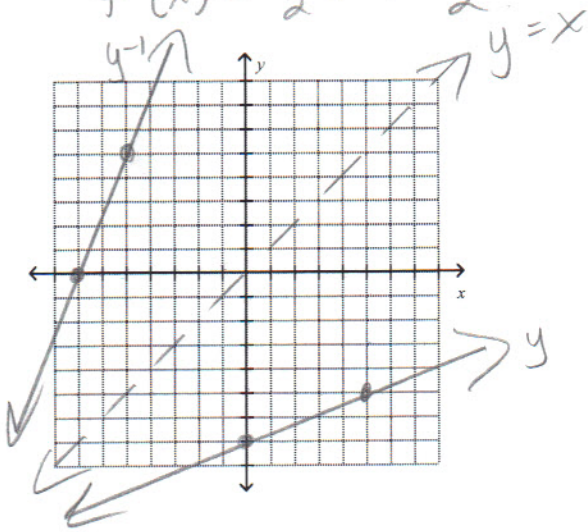
$x = \frac{2}{5}y - 7$

$x + 7 = \frac{2}{5}y$

$5x + 35 = 2y$

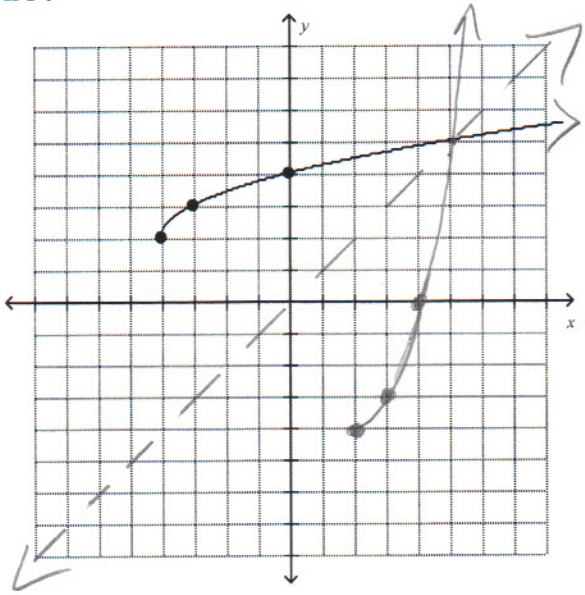
$\frac{5}{2}x + \frac{35}{2} = y$

$f^{-1}(x) = \frac{5}{2}x + \frac{35}{2}$



The graph of a relation is given. Graph the inverse.

15.



16.