

C6L5 Notes

Solving Radical and Rational Exponent Equations

Solve.

1. $(1 - 16n)^{\frac{1}{4}} = 3$

$$\left((1-16n)^{\frac{1}{4}}\right)^{\frac{4}{1}} = 3^{\frac{4}{1}}$$

$$\begin{array}{r} 1-16n = 81 \\ -1 \qquad -1 \end{array}$$

$$-16n = 80$$

$$n = -5$$

2. $(n - 5)^{\frac{3}{2}} + 3 = 67$

$$\begin{array}{r} (n-5)^{\frac{3}{2}} + 3 = 67 \\ -3 \quad -3 \end{array}$$

$$(n-5)^{\frac{3}{2}} = 64$$

$$\left((n-5)^{\frac{3}{2}}\right)^{\frac{2}{3}} = 64^{\frac{2}{3}}$$

$$n-5 = 16$$

$$n = 21$$

3. $-131 = 4 - 5(2x - 7)^{\frac{3}{2}}$

$$\begin{array}{r} 4 - 5(2x-7)^{\frac{3}{2}} = -131 \\ -4 \qquad -4 \end{array}$$

$$-5(2x-7)^{\frac{3}{2}} = -135$$

$$(2x-7)^{\frac{3}{2}} = 27$$

$$\left((2x-7)^{\frac{3}{2}}\right)^{\frac{2}{3}} = 27^{\frac{2}{3}}$$

$$2x-7 = 9$$

$$2x = 16$$

$$x = 8$$

4. $151 = 7 + 4(62 - n)^{\frac{2}{3}}$

$$\begin{array}{r} 7 + 4(62-n)^{\frac{2}{3}} = 151 \\ -7 \qquad -7 \end{array}$$

$$4(62-n)^{\frac{2}{3}} = 144$$

$$(62-n)^{\frac{2}{3}} = 36$$

$$\left((62-n)^{\frac{2}{3}}\right)^{\frac{3}{2}} = \pm 36^{\frac{3}{2}}$$

$$62-n = \pm 216$$

$$62-n = 216$$

$$-n = 154$$

$$n = -154$$

$$62-n = -216$$

$$-n = -278$$

$$n = 278$$

$$5. 3(5x+2)^{\frac{4}{5}} - 5 = 43$$

$$3(5x+2)^{\frac{4}{5}} = 48$$

$$(5x+2)^{\frac{4}{5}} = 16$$

$$\left((5x+2)^{\frac{4}{5}}\right)^{\frac{5}{4}} = \pm 16^{\frac{5}{4}}$$

$$5x+2 = \pm 32$$

$$5x+2=32 \quad 5x+2=-32$$

$$5x=30 \quad 5x=-34$$

$$x=6 \quad x=\frac{-34}{5}$$

$$x=6, \frac{-34}{5}$$

$$6. (3x+7)^{\frac{5}{6}} = (17-5x)^{\frac{5}{6}}$$

$$\left(\left(3x+7\right)^{\frac{5}{6}}\right)^{\frac{6}{5}} = \left(\left(17-5x\right)^{\frac{5}{6}}\right)^{\frac{6}{5}}$$

$$3x+7 = 17-5x$$

$$\begin{array}{r} +5x \qquad \qquad \qquad +5x \\ \hline \end{array}$$

$$8x+7 = 17$$

$$\begin{array}{r} -7 \quad -7 \\ \hline \end{array}$$

$$8x = 10$$

$$x = \frac{10}{8}$$

$$x = \frac{5}{4}$$

$$7. -3 = \sqrt{2x-7} - 10$$

$$\sqrt{2x-7} - 10 = -3$$

$$(2x-7)^{\frac{1}{2}} - 10 = -3$$

$$(2x-7)^{\frac{1}{2}} = 7$$

$$\left((2x-7)^{\frac{1}{2}}\right)^2 = 7^2$$

$$2x-7 = 49$$

$$2x = 56$$

$$x = 28$$

$$8. 5\sqrt{3x-5} + 4 = 39$$

$$5(3x-5)^{\frac{1}{2}} + 4 = 39$$

$$5(3x-5)^{\frac{1}{2}} = 35$$

$$(3x-5)^{\frac{1}{2}} = 7$$

$$\left((3x-5)^{\frac{1}{2}}\right)^2 = 7^2$$

$$3x-5 = 49$$

$$3x = 54$$

$$x = 18$$

$$9. x = \sqrt{30 - x}$$

$$X = (30 - X)^{\frac{1}{2}}$$

$$X^2 = ((30 - X)^{\frac{1}{2}})^2$$

$$X^2 = 30 - X$$

$$\begin{array}{r} +X \quad +X \\ \hline X^2 + X = 30 \\ -30 \quad -30 \\ \hline \end{array}$$

$$\begin{aligned} X^2 + X - 30 &= 0 \\ (X + 6)(X - 5) &= 0 \\ X + 6 = 0 \quad X - 5 = 0 \end{aligned}$$

$$X = \cancel{-6}, 5$$

- BUT, IF $X = -6$, THE EQUATION IS UNDEFINED. THEREFORE, THE ONLY SOLUTION IS

$$X = 5.$$

$$11. 5x = \sqrt{30x + 40}$$

$$5x = (30x + 40)^{\frac{1}{2}}$$

$$(5x)^2 = ((30x + 40)^{\frac{1}{2}})^2$$

$$25x^2 = 30x + 40$$

$$\begin{array}{r} -30x \quad -30x \\ \hline 25x^2 - 30x = 40 \\ -40 \quad -40 \\ \hline \end{array}$$

$$\frac{25x^2}{5} - \frac{30x}{5} - \frac{40}{5} = 0$$

$$5(5x^2 - 6x - 8) = 0$$

$$5(5x + 4)(x - 2) = 0$$

$$\begin{array}{r} 5x + 4 = 0 \quad x - 2 = 0 \\ -4 \quad -4 \quad +2 \quad +2 \\ \hline 5x = -4 \quad x = 2 \\ x = \cancel{-\frac{4}{5}} \end{array}$$

$$X = 2$$

$$10. \sqrt{56 - x} = x$$

$$(56 - x)^{\frac{1}{2}} = x$$

$$((56 - x)^{\frac{1}{2}})^2 = x^2$$

$$56 - x = x^2$$

$$\begin{array}{r} +x \quad +x \\ \hline 56 = x^2 + x \\ -56 \quad -56 \\ \hline \end{array}$$

$$0 = x^2 + x - 56$$

$$0 = (x + 8)(x - 6)$$

$$x + 8 = 0 \quad x - 6 = 0$$

$$x = \cancel{-8}, 6$$

$$12. 7x = \sqrt{77x + 42}$$

$$7x = (77x + 42)^{\frac{1}{2}}$$

$$(7x)^2 = ((77x + 42)^{\frac{1}{2}})^2$$

$$49x^2 = 77x + 42$$

$$\begin{array}{r} -77x \quad -77x \\ \hline 49x^2 - 77x = 42 \\ -42 \quad -42 \\ \hline \end{array}$$

$$\frac{49x^2}{7} - \frac{77x}{7} - \frac{42}{7} = 0$$

$$7(7x^2 - 11x - 6) = 0$$

$$7(7x + 3)(x - 2) = 0$$

$$7x + 3 = 0$$

$$x - 2 = 0$$

$$\begin{array}{r} -3 \quad -3 \\ \hline 7x = -3 \\ x = \cancel{-\frac{3}{7}} \end{array}$$

$$\begin{array}{r} +2 \quad +2 \\ \hline x = 2 \end{array}$$

$$X = 2$$

$$13. n = \sqrt{8n - 12}$$

$$n = (8n - 12)^{\frac{1}{2}}$$

$$n^2 = ((8n - 12)^{\frac{1}{2}})^2$$

$$n^2 = 8n - 12$$

$$\begin{array}{r} n^2 - 8n = -12 \\ +12 \quad +12 \\ \hline \end{array}$$

$$n^2 - 8n + 12 = 0$$

$$(n - 6)(n - 2) = 0$$

$$n - 6 = 0 \quad n - 2 = 0$$

$$n = 6, 2$$

$$14. m = \sqrt{-6m - 9}$$

$$m = (-6m - 9)^{\frac{1}{2}}$$

$$m^2 = ((-6m - 9)^{\frac{1}{2}})^2$$

$$m^2 = -6m - 9$$

$$\begin{array}{r} m^2 + 6m = -9 \\ +6m \quad +6m \\ \hline \end{array}$$

$$m^2 + 6m + 9 = 0$$

$$(m + 3)(m + 3) = 0$$

$$(m + 3)(m + 3) = 0$$

$$m + 3 = 0$$

$$m + 3 = 0$$

$$m = -3$$

$$m = -3$$

NO SOLUTION

$$15. 15 = 12 + \sqrt[3]{4x - 5}$$

$$12 + \sqrt[3]{4x - 5} = 15$$

$$12 + (4x - 5)^{\frac{1}{3}} = 15$$

$$\begin{array}{r} -12 \quad -12 \\ \hline \end{array}$$

$$(4x - 5)^{\frac{1}{3}} = 3$$

$$((4x - 5)^{\frac{1}{3}})^3 = 3^3$$

$$4x - 5 = 27$$

$$\begin{array}{r} +5 \quad +5 \\ \hline \end{array}$$

$$4x = 32$$

$$x = 8$$

$$16. -7 = \sqrt[4]{6 - 11n} - 9$$

$$\sqrt[4]{6 - 11n} - 9 = -7$$

$$(6 - 11n)^{\frac{1}{4}} - 9 = -7$$

$$\begin{array}{r} +9 \quad +9 \\ \hline \end{array}$$

$$(6 - 11n)^{\frac{1}{4}} = 2$$

$$((6 - 11n)^{\frac{1}{4}})^4 = 2^4$$

$$6 - 11n = 16$$

$$\begin{array}{r} -6 \quad -6 \\ \hline \end{array}$$

$$-11n = 10$$

$$n = \frac{-10}{11}$$

$$17. 4\sqrt[5]{(9n-4)^3} - 18 = 90$$

$$4(9n-4)^{\frac{3}{5}} - 18 = 90$$

$$\begin{array}{r} +18 \quad +18 \\ \hline \end{array}$$

$$\frac{4(9n-4)^{\frac{3}{5}}}{4} = \frac{108}{4}$$

$$(9n-4)^{\frac{3}{5}} = 27$$

$$\left((9n-4)^{\frac{3}{5}}\right)^{\frac{5}{3}} = 27^{\frac{5}{3}}$$

$$9n-4 = 243$$

$$\begin{array}{r} +4 \quad +4 \\ \hline \end{array}$$

$$9n = 247$$

$$n = \frac{247}{9}$$

$$18. 2\sqrt[5]{(6n+7)^2} - 58 = 14$$

$$2(6n+7)^{\frac{2}{5}} - 58 = 14$$

$$\begin{array}{r} +58 \quad +58 \\ \hline \end{array}$$

$$\frac{2(6n+7)^{\frac{2}{5}}}{2} = \frac{72}{2}$$

$$(6n+7)^{\frac{2}{5}} = 36$$

$$\left((6n+7)^{\frac{2}{5}}\right)^{\frac{5}{2}} = \pm 36^{\frac{5}{2}}$$

$$6n+7 = \pm 7776$$

$$6n+7 = 7776 \quad \text{OR} \quad 6n+7 = -7776$$

$$\begin{array}{r} -7 \quad -7 \\ \hline \end{array}$$

$$6n = 7769$$

$$6n = -7783$$

$$n = \frac{7769}{6}, \frac{-7783}{6}$$