

C6L3 Notes

Operations with Radical Expressions

Simplify.

1. $7\sqrt[4]{3} - 2\sqrt[4]{3}$

$5\sqrt[4]{3}$

2. $5\sqrt{y} - \sqrt{y}$

$4\sqrt{y}$

3. $12\sqrt{45} - 9\sqrt{125}$

$12\sqrt{9 \cdot 5} - 9\sqrt{25 \cdot 5}$
 $12\sqrt{9} \cdot \sqrt{5} - 9\sqrt{25} \cdot \sqrt{5}$
 $12 \cdot 3\sqrt{5} - 9 \cdot 5\sqrt{5}$
 $36\sqrt{5} - 45\sqrt{5}$
 $-9\sqrt{5}$

$\begin{array}{r} 5 \overline{)45} \\ \underline{30} \\ 15 \end{array}$

$\begin{array}{r} 5 \overline{)125} \\ \underline{50} \\ 75 \\ \underline{75} \\ 0 \end{array}$

4. $6\sqrt[3]{54} - \sqrt[3]{16}$

$6\sqrt[3]{27 \cdot 2} - \sqrt[3]{8 \cdot 2}$
 $6\sqrt[3]{27} \cdot \sqrt[3]{2} - \sqrt[3]{8} \cdot \sqrt[3]{2}$
 $6 \cdot 3\sqrt[3]{2} - 2\sqrt[3]{2}$
 $18\sqrt[3]{2} - 2\sqrt[3]{2}$
 $16\sqrt[3]{2}$

$\begin{array}{r} 2 \overline{)54} \\ \underline{36} \\ 18 \end{array}$

$\begin{array}{r} 2 \overline{)16} \\ \underline{12} \\ 4 \\ \underline{4} \\ 0 \end{array}$

5. $6\sqrt{18x} + 7\sqrt{98x}$

$6\sqrt{9 \cdot 2 \cdot x} + 7\sqrt{49 \cdot 2 \cdot x}$
 $6\sqrt{9} \sqrt{2} \sqrt{x} + 7\sqrt{49} \sqrt{2} \sqrt{x}$
 $6 \cdot 3\sqrt{2x} + 7 \cdot 7\sqrt{2x}$
 $18\sqrt{2x} + 49\sqrt{2x}$
 $67\sqrt{2x}$

$\begin{array}{r} 2 \overline{)18} \\ \underline{12} \\ 6 \end{array}$

$\begin{array}{r} 2 \overline{)98} \\ \underline{70} \\ 28 \end{array}$

6. $\sqrt{75} + 2\sqrt{48} - 6\sqrt{12}$

$\sqrt{25 \cdot 3} + 2\sqrt{4 \cdot 4 \cdot 3} - 6\sqrt{4 \cdot 3}$
 $\sqrt{25} \sqrt{3} + 2\sqrt{4} \sqrt{4} \sqrt{3} - 6\sqrt{4} \sqrt{3}$
 $5\sqrt{3} + 2 \cdot 2 \cdot 2\sqrt{3} - 6 \cdot 2\sqrt{3}$
 $5\sqrt{3} + 8\sqrt{3} - 12\sqrt{3}$
 $\sqrt{3}$

$\begin{array}{r} 3 \overline{)75} \\ \underline{50} \\ 25 \end{array}$

$\begin{array}{r} 2 \overline{)48} \\ \underline{24} \\ 24 \\ \underline{24} \\ 0 \end{array}$

$\begin{array}{r} 2 \overline{)12} \\ \underline{12} \\ 0 \end{array}$

7. $\sqrt[3]{375} + \sqrt[3]{1029} - \sqrt[3]{192}$

$\sqrt[3]{125 \cdot 3} + \sqrt[3]{343 \cdot 3} - \sqrt[3]{8 \cdot 8 \cdot 3}$
 $\sqrt[3]{125} \cdot \sqrt[3]{3} + \sqrt[3]{343} \cdot \sqrt[3]{3} - \sqrt[3]{8} \cdot \sqrt[3]{8} \cdot \sqrt[3]{3}$
 $5\sqrt[3]{3} + 7\sqrt[3]{3} - 2 \cdot 2 \cdot \sqrt[3]{3}$
 $5\sqrt[3]{3} + 7\sqrt[3]{3} - 4\sqrt[3]{3}$
 $8\sqrt[3]{3}$

$\begin{array}{r} 3 \overline{)375} \\ \underline{150} \\ 225 \\ \underline{225} \\ 0 \end{array}$

$\begin{array}{r} 3 \overline{)1029} \\ \underline{700} \\ 329 \\ \underline{329} \\ 0 \end{array}$

$\begin{array}{r} 2 \overline{)192} \\ \underline{160} \\ 32 \\ \underline{32} \\ 0 \end{array}$

Multiply, then simplify.

8. $(4 + \sqrt{11})(3 - 2\sqrt{11})$

$$12 - 8\sqrt{11} + 3\sqrt{11} - 2\sqrt{121}$$

$$12 - 5\sqrt{11} - 2 \cdot 11$$

$$12 - 5\sqrt{11} - 22$$

$$-10 - 5\sqrt{11}$$

9. $(2 - 3\sqrt{7})(6 - 5\sqrt{7})$

$$12 - 10\sqrt{7} - 18\sqrt{7} + 15\sqrt{49}$$

$$12 - 28\sqrt{7} + 15 \cdot 7$$

$$12 - 28\sqrt{7} + 105$$

$$117 - 28\sqrt{7}$$

10. $(\sqrt{2} + \sqrt{5})^2$

$$(\sqrt{2} + \sqrt{5})(\sqrt{2} + \sqrt{5})$$

$$\sqrt{4} + \sqrt{10} + \sqrt{10} + \sqrt{25}$$

$$2 + 2\sqrt{10} + 5$$

$$7 + 2\sqrt{10}$$

11. $(4\sqrt{3} - 2\sqrt{7})^2$

$$(4\sqrt{3} - 2\sqrt{7})(4\sqrt{3} - 2\sqrt{7})$$

$$16\sqrt{9} - 8\sqrt{21} - 8\sqrt{21} + 4\sqrt{49}$$

$$16 \cdot 3 - 16\sqrt{21} + 4 \cdot 7$$

$$48 - 16\sqrt{21} + 28$$

$$76 - 16\sqrt{21}$$

12. $(6 - 5\sqrt{2})(6 + 5\sqrt{2})$

$$36 + 30\sqrt{2} - 30\sqrt{2} - 25\sqrt{4}$$

$$36 - 25 \cdot 2$$

$$36 - 50$$

$$-14$$

13. $(5 + \sqrt{3})(5 - \sqrt{3})$

$$25 - 5\sqrt{3} + 5\sqrt{3} - \sqrt{9}$$

$$25 - 3$$

$$22$$

14. $(1 + \sqrt{50})(5 + \sqrt{2})$

$$5 + 1\sqrt{2} + 5\sqrt{50} + \sqrt{100}$$

$$5 + 1\sqrt{2} + 5\sqrt{50} + 10$$

$$15 + 1\sqrt{2} + 5\sqrt{25 \cdot 2}$$

$$15 + 1\sqrt{2} + 5\sqrt{25}\sqrt{2}$$

$$15 + 1\sqrt{2} + 5 \cdot 5\sqrt{2}$$

$$15 + 1\sqrt{2} + 25\sqrt{2}$$

$$15 + 26\sqrt{2}$$

15. $(\sqrt{x} + \sqrt{3})(\sqrt{x} - 4\sqrt{3})$

$$\sqrt{x^2} - 4\sqrt{3x} + \sqrt{3x} - 4\sqrt{9}$$

$$|x| - 3\sqrt{3x} - 4 \cdot 3$$

$$|x| - 3\sqrt{3x} - 12$$

Rationalize the denominator, then simplify.

$$\begin{aligned}
 16. \quad & \frac{5}{1-\sqrt{3}} \cdot \frac{1+\sqrt{3}}{1+\sqrt{3}} \\
 & \frac{5(1+\sqrt{3})}{(1-\sqrt{3})(1+\sqrt{3})} \\
 & \frac{5+5\sqrt{3}}{1+\sqrt{3}-\sqrt{3}-\sqrt{9}} \\
 & \frac{5+5\sqrt{3}}{1-3} \\
 & \frac{5+5\sqrt{3}}{-2}
 \end{aligned}$$

$$\begin{aligned}
 17. \quad & \frac{2+\sqrt{7}}{3-\sqrt{7}} \cdot \frac{3+\sqrt{7}}{3+\sqrt{7}} \\
 & \frac{(2+\sqrt{7})(3+\sqrt{7})}{(3-\sqrt{7})(3+\sqrt{7})} \\
 & \frac{6+2\sqrt{7}+3\sqrt{7}+\sqrt{49}}{9+3\sqrt{7}-3\sqrt{7}-\sqrt{49}} \\
 & \frac{6+5\sqrt{7}+7}{9-7} \\
 & \frac{13+5\sqrt{7}}{2}
 \end{aligned}$$

$$\begin{aligned}
 18. \quad & \frac{3+\sqrt{8}}{4-3\sqrt{8}} \cdot \frac{4+3\sqrt{8}}{4+3\sqrt{8}} \\
 & \frac{(3+\sqrt{8})(4+3\sqrt{8})}{(4-3\sqrt{8})(4+3\sqrt{8})} \\
 & \frac{12+9\sqrt{8}+4\sqrt{8}+3\sqrt{64}}{16+12\sqrt{8}-12\sqrt{8}-9\sqrt{64}} \\
 & \frac{12+13\sqrt{8}+3 \cdot 8}{16-9 \cdot 8} \\
 & \frac{12+13\sqrt{8}+24}{16-72} \\
 & \frac{36+13\sqrt{8}}{-56} \\
 & \frac{36+13\sqrt{4 \cdot 2}}{-56} \\
 & \frac{36+26\sqrt{2}}{-56} = \frac{18+13\sqrt{2}}{-28}
 \end{aligned}$$

$$\begin{aligned}
 19. \quad & \frac{4-\sqrt{35}}{\sqrt{5}-\sqrt{7}} \cdot \frac{\sqrt{5}+\sqrt{7}}{\sqrt{5}+\sqrt{7}} \\
 & \frac{(4-\sqrt{35})(\sqrt{5}+\sqrt{7})}{(\sqrt{5}-\sqrt{7})(\sqrt{5}+\sqrt{7})} \\
 & \frac{4\sqrt{5}+4\sqrt{7}-\sqrt{175}-\sqrt{245}}{\sqrt{25}+\sqrt{35}-\sqrt{35}-\sqrt{49}} \\
 & \frac{4\sqrt{5}+4\sqrt{7}-5\sqrt{7}-7\sqrt{5}}{5-7} \\
 & \frac{-3\sqrt{5}-\sqrt{7}}{-2} \\
 & \frac{3\sqrt{5}+\sqrt{7}}{2}
 \end{aligned}$$