8.4 Assess Your Understanding

'Are You Prepared?' *The answer is given at the end of these exercises. If you get the wrong answer, read the pages listed in red.*

1. The area K of a triangle, whose base is b and whose height is h is ________. (p. A15)

Concepts and Vocabulary

2. If two sides a and b and the included angle C are known in a triangle, then the area K is found using the formula

$K =$ ________.

3. The area K of a triangle with sides a, b, and c is

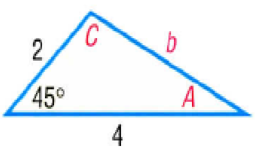
$K =$ __________________ where $s =$ __________.

4. ***True or False*** Heron's Formula is used to find the area of SSS triangles.

Skill Building

In Problems 5–12, find the area of each triangle. Round answers to two decimal places.

5.



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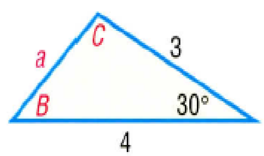

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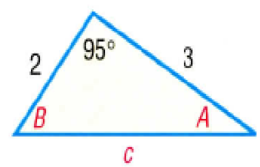

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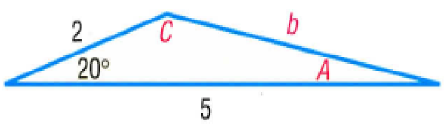

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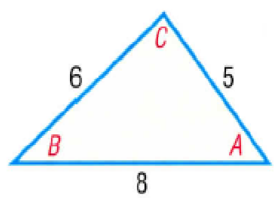

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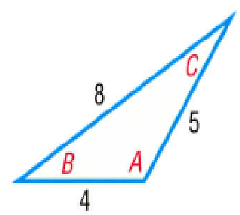

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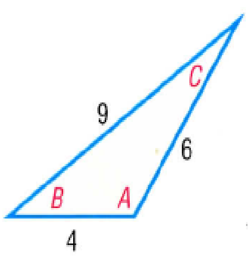

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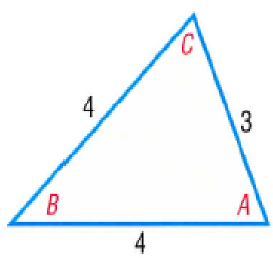


In Problems 13–24, find the area of each triangle. Round answers to two decimal places.

13. $a = 3, \quad b = 4, \quad C = 40°$

14. $a = 2, \quad c = 1, \quad B = 10°$

15. $b = 1, \quad c = 3, \quad A = 80°$

16. $a = 6, \quad b = 4, \quad C = 60°$

17. $a = 3, \quad c = 2, \quad B = 110°$

18. $b = 4, \quad c = 1, \quad A = 120°$

19. $a = 12, \quad b = 13, \quad c = 5$

20. $a = 4, \quad b = 5, \quad c = 3$

21. $a = 2, \quad b = 2, \quad c = 2$

22. $a = 3, \quad b = 3, \quad c = 2$

23. $a = 5, \quad b = 8, \quad c = 9$

24. $a = 4, \quad b = 3, \quad c = 6$

Applications and Extensions

25. **Area of an ASA Triangle** If two angles and the included side are given, the third angle is easy to find. Use the Law of Sines to show that the area K of a triangle with side a and angles A, B, and C is

$$K = \frac{a^2 \sin B \sin C}{2 \sin A}$$

26. **Area of a Triangle** Prove the two other forms of the formula given in Problem 25.

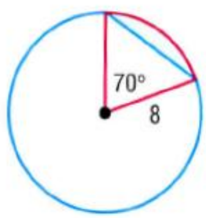
$$K = \frac{b^2 \sin A \sin C}{2 \sin B} \quad \text{and} \quad K = \frac{c^2 \sin A \sin B}{2 \sin C}$$

In Problems 27–32, use the results of Problem 25 or 26 to find the area of each triangle. Round answers to two decimal places.

27. $A = 40°, \ B = 20°, \ a = 2$
28. $A = 50°, \ C = 20°, \ a = 3$
29. $B = 70°, \ C = 10°, \ b = 5$
30. $A = 70°, \ B = 60°, \ c = 4$
31. $A = 110°, \ C = 30°, \ c = 3$
32. $B = 10°, \ C = 100°, \ b = 2$

33. **Area of a Segment** Find the area of the segment (shaded in blue in the figure) of a circle whose radius is 8 feet, formed by a central angle of 70°.

[**Hint:** Subtract the area of the triangle from the area of the sector to obtain the area of the segment.]

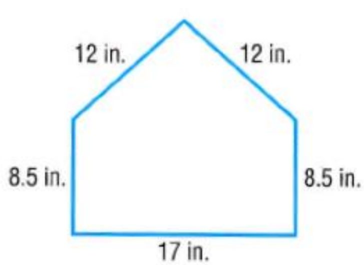


34. **Area of a Segment** Find the area of the segment of a circle whose radius is 5 inches, formed by a central angle of 40°.

35. **Cost of a Triangular Lot** The dimensions of a triangular lot are 100 feet by 50 feet by 75 feet. If the price of such land is $3 per square foot, how much does the lot cost?

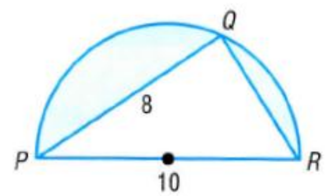
36. **Amount of Material to Make a Tent** A cone-shaped tent is made from a circular piece of canvas 24 feet in diameter by removing a sector with central angle 100° and connecting the ends. What is the surface area of the tent?

37. **Dimensions of Home Plate** The dimensions of home plate at any major league baseball stadium are shown. Find the area of home plate.

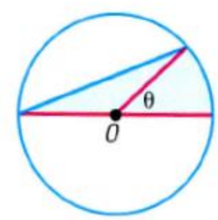


38. **Computing Areas** See the following figure. Find the area of the shaded region enclosed in a semicircle of diameter 10 inches. The length of the chord PQ is 8 inches.

[**Hint:** Triangle PQR is a right triangle.]

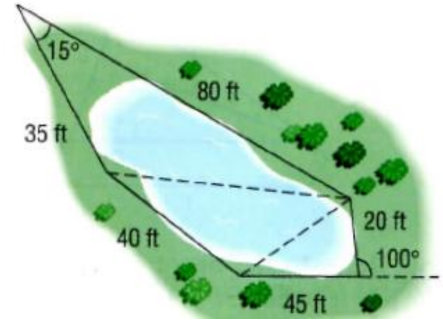


39. **Geometry** Consult the figure, which shows a circle of radius r with center at O. Find the area K of the shaded region as a function of the central angle θ.



40. **Approximating the Area of a Lake** To approximate the area of a lake, a surveyor walks around the perimeter of the lake, taking the measurements shown in the illustration. Using this technique, what is the approximate area of the lake?

[**Hint:** Use the Law of Cosines on the three triangles shown and then find the sum of their areas.]



41. **The Flatiron Building** Completed in 1902 in New York City, the Flatiron Building is triangular shaped and bounded by 22nd Street, Broadway, and 5th Avenue. The building

measures approximately 87 feet on the 22nd Street side, 190 feet on the Broadway side, and 173 feet on the 5th Avenue side. Approximate the ground area covered by the building.

Source: Sarah Bradford Landau and Carl W. Condit, *Rise of the New York Skyscraper: 1865–1913.* New Haven, CT: Yale University Press, 1996

42. Bermuda Triangle The Bermuda Triangle is roughly defined by Hamilton, Bermuda; San Juan, Puerto Rico; and Fort Lauderdale, Florida. The distances from Hamilton to Fort Lauderdale, Fort Lauderdale to San Juan, and San Juan to Hamilton are approximately 1028, 1046, and 965 miles, respectively. Ignoring the curvature of Earth, approximate the area of the Bermuda Triangle.

Source: *www.worldatlas.com*

43. Geometry Refer to the figure. If $|OA| = 1$, show that:

(a) Area $\Delta OAC = \frac{1}{2}\sin\alpha\cos\alpha$

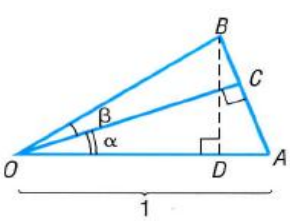
(b) Area $\Delta OCB = \frac{1}{2}|OB|^2\sin\beta\cos\beta$

(c) Area $\Delta OAB = \frac{1}{2}|OB|\sin(\alpha+\beta)$

(d) $|OB| = \dfrac{\cos\alpha}{\cos\beta}$

(e) $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

[**Hint:** Area ΔOAB = Area ΔOAC + Area ΔOCB.]



44. Geometry Refer to the figure, in which a unit circle is drawn. The line segment DB is tangent to the circle and θ is acute.

(a) Express the area of ΔOBC in terms of $\sin\theta$ and $\cos\theta$.

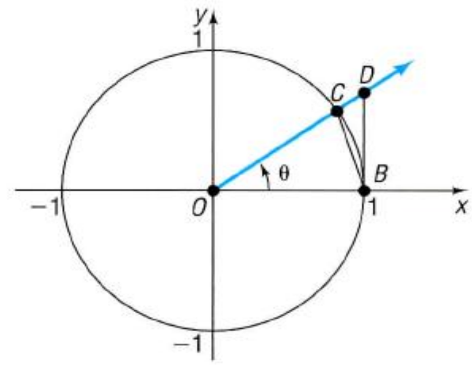
(b) Express the area of ΔOBD in terms of $\sin\theta$ and $\cos\theta$.

(c) The area of the sector $\widehat{OBC}$ of the circle is $\frac{1}{2}\theta$, where θ is measured in radians. Use the results of parts (a) and (b) and the fact that

$$\text{Area } \Delta OBC < \text{Area } \widehat{OBC} < \text{Area } \Delta OBD$$

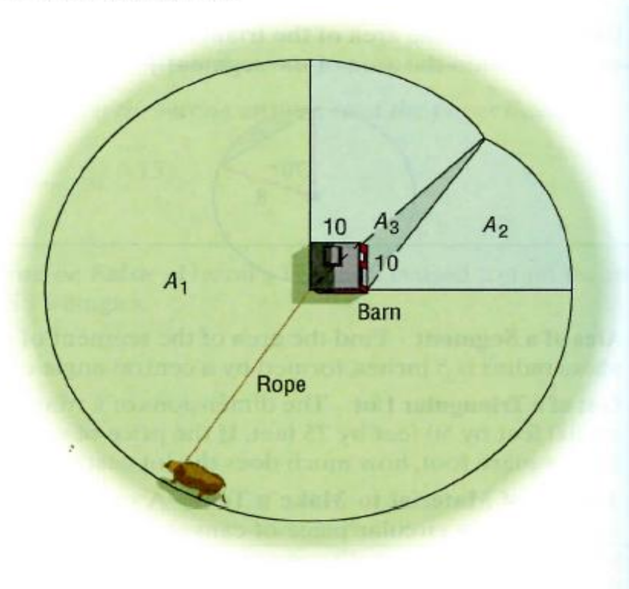
to show that

$$1 < \frac{\theta}{\sin\theta} < \frac{1}{\cos\theta}$$



45. The Cow Problem* A cow is tethered to one corner of a square barn, 10 feet by 10 feet, with a rope 100 feet long. What is the maximum grazing area for the cow?

[See the illustration.]



46. Another Cow Problem If the barn in Problem 45 is rectangular, 10 feet by 20 feet, what is the maximum grazing area for the cow?

47. Perfect Triangles A *perfect triangle* is one having natural number sides for which the area is numerically equal to the perimeter. Show that the triangles with the given side lengths are perfect.

(a) 9, 10, 17 (b) 6, 25, 29

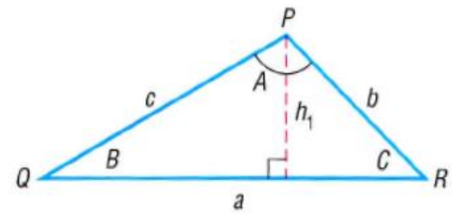
Source: M.V. Bonsangue, G. E. Gannon, E. Buchman, and N. Gross, "In Search of Perfect Triangles," *Mathematics Teacher,* Vol. 92, No. 1, 1999: 56–61

*Suggested by Professor Teddy Koukounas of Suffolk Community College, who learned of it from an old farmer in Virginia. Solution provided by Professor Kathleen Miranda of SUNY at Old Westbury.

48. If h_1, h_2, and h_3 are the altitudes dropped from P, Q, and R, respectively, in a triangle (see the figure shown to the right), show that

$$\frac{1}{h_1}+\frac{1}{h_2}+\frac{1}{h_3}=\frac{s}{K}$$

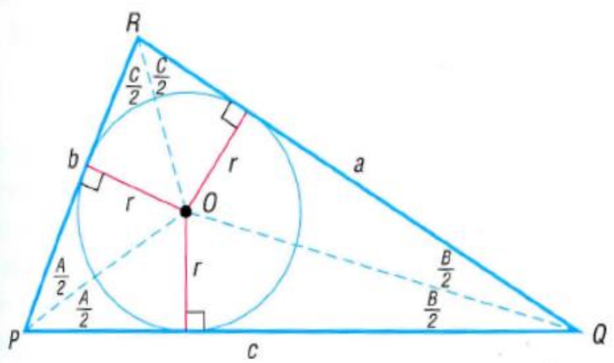
where K is the area of the triangle and $s=\frac{1}{2}(a+b+c)$. [**Hint:** $h_1=\frac{2K}{a}$.]



49. Show that a formula for the altitude h from a vertex to the opposite side a of a triangle is

$$h=\frac{a\sin B\sin C}{\sin A}$$

Inscribed Circle *For Problems 50–53, the lines that bisect each angle of a triangle meet in a single point O, and the perpendicular distance r from O to each side of the triangle is the same. The circle with center at O and radius r is called the* ***inscribed circle*** *of the triangle (see the figure).*



50. Apply the formula from Problem 49 to triangle OPQ to show that

$$r=\frac{c\sin\frac{A}{2}\sin\frac{B}{2}}{\cos\frac{C}{2}}$$

51. Use the result of Problem 50 and the results of Problems 56 and 57 in Section 8.3 to show that

$$\cot\frac{C}{2}=\frac{s-c}{r}$$

where $s=\frac{1}{2}(a+b+c)$.

52. Show that

$$\cot\frac{A}{2}+\cot\frac{B}{2}+\cot\frac{C}{2}=\frac{s}{r}$$

53. Show that the area K of triangle PQR is $K=rs$, where $s=\frac{1}{2}(a+b+c)$. Then show that

$$r=\sqrt{\frac{(s-a)(s-b)(s-c)}{s}}$$

Explaining Concepts: Discussion and Writing

54. What do you do first if you are asked to find the area of a triangle and are given two sides and the included angle?

55. What do you do first if you are asked to find the area of a triangle and are given three sides?

56. State the formula for the area of an SAS triangle in words.

'Are You Prepared?' Answer

1. $K=\frac{1}{2}bh$