8.3 Assess Your Understanding

'Are You Prepared?' *Answers are given at the end of these exercises. If you get a wrong answer, read the pages listed in red.*

1. Write the formula for the distance d from $P_1 = (x_1, y_1)$ to $P_2 = (x_2, y_2)$. (p. 5)

2. If θ is an acute angle, solve the equation $\cos\theta = \frac{\sqrt{2}}{2}$. (pp. 459–461)

Concepts and Vocabulary

3. If three sides of a triangle are given, the Law of _____ is used to solve the triangle.

4. If one side and two angles of a triangle are given, the Law of _____ is used to solve the triangle.

5. If two sides and the included angle of a triangle are given, the Law of _____ is used to solve the triangle.

6. ***True or False*** Given only the three sides of a triangle, there is insufficient information to solve the triangle.

7. ***True or False*** Given two sides and the included angle, the first thing to do to solve the triangle is to use the Law of Sines.

8. ***True or False*** A special case of the Law of Cosines is the Pythagorean Theorem.

Skill Building

In Problems 9–16, solve each triangle.

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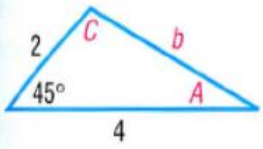

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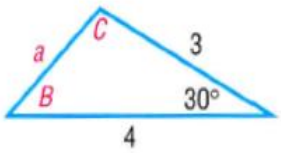

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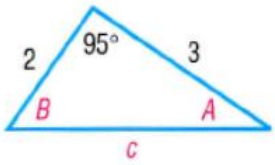

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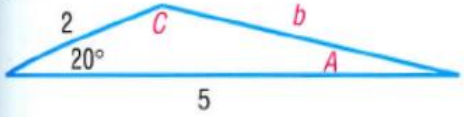

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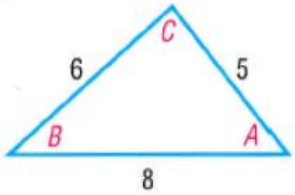

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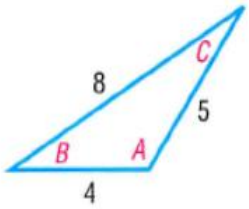

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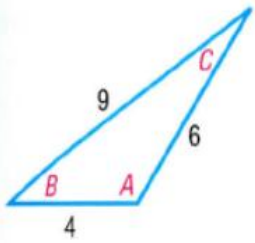

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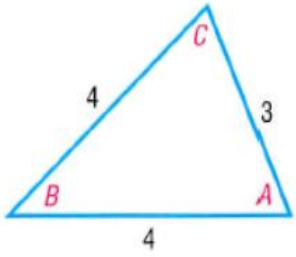


In Problems 17–32, solve each triangle.

17. $a = 3, \quad b = 4, \quad C = 40°$
18. $a = 2, \quad c = 1, \quad B = 10°$
19. $b = 1, \quad c = 3, \quad A = 80°$
20. $a = 6, \quad b = 4, \quad C = 60°$
21. $a = 3, \quad c = 2, \quad B = 110°$
22. $b = 4, \quad c = 1, \quad A = 120°$
23. $a = 2, \quad b = 2, \quad C = 50°$
24. $a = 3, \quad c = 2, \quad B = 90°$
25. $a = 12, \quad b = 13, \quad c = 5$
26. $a = 4, \quad b = 5, \quad c = 3$
27. $a = 2, \quad b = 2, \quad c = 2$
28. $a = 3, \quad b = 3, \quad c = 2$
29. $a = 5, \quad b = 8, \quad c = 9$
30. $a = 4, \quad b = 3, \quad c = 6$
31. $a = 10, \quad b = 8, \quad c = 5$
32. $a = 9, \quad b = 7, \quad c = 10$

Mixed Practice

In Problems 33–42, solve each triangle using either the Law of Sines or the Law of Cosines.

33. $B = 20°, C = 75°, b = 5$

34. $A = 50°, B = 55°, c = 9$

35. $a = 6, b = 8, c = 9$

36. $a = 14, b = 7, A = 85°$

37. $B = 35°, C = 65°, a = 15$

38. $a = 4, c = 5, B = 55°$

39. $A = 10°, a = 3, b = 10$

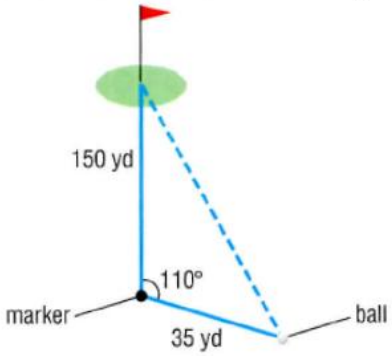
40. $A = 65°, B = 72°, b = 7$

41. $b = 5, c = 12, A = 60°$

42. $a = 10, b = 10, c = 15$

Applications and Extensions

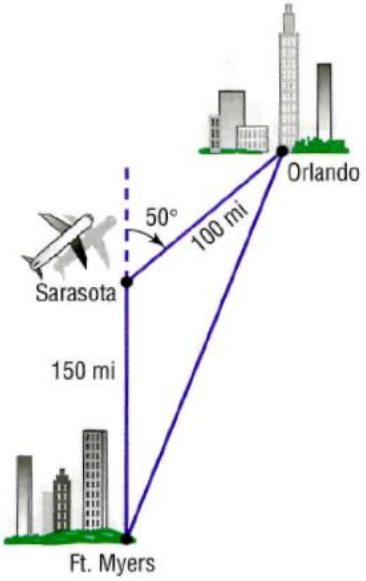
43. Distance to the Green A golfer hits an errant tee shot that lands in the rough. A marker in the center of the fairway is 150 yards from the center of the green. While standing on the marker and facing the green, the golfer turns 110° toward his ball. He then paces off 35 yards to his ball. See the figure. How far is the ball from the center of the green?



44. Navigation An airplane flies due north from Ft. Myers to Sarasota, a distance of 150 miles and then turns through an angle of 50° and flies to Orlando, a distance of 100 miles. See the figure.

(a) How far is it directly from Ft. Myers to Orlando?

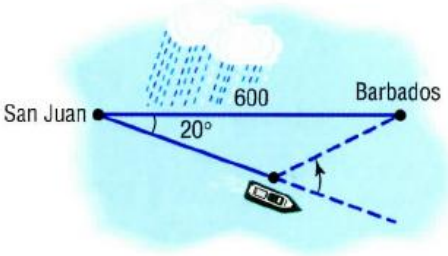
(b) What bearing should the pilot use to fly directly from Ft. Myers to Orlando?



45. Avoiding a Tropical Storm A cruise ship maintains an average speed of 15 knots in going from San Juan, Puerto Rico, to Barbados, West Indies, a distance of 600 nautical miles. To avoid a tropical storm, the captain heads out of San Juan in a direction of 20° off a direct heading to Barbados. The captain maintains the 15-knot speed for 10 hours, after which time the path to Barbados becomes clear of storms.

(a) Through what angle should the captain turn to head directly to Barbados?

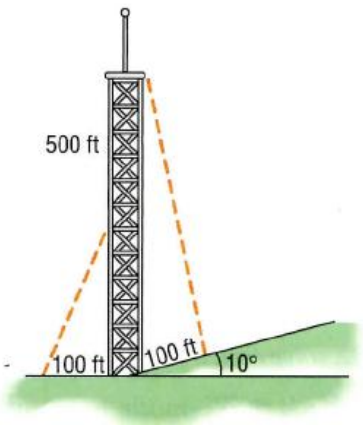
(b) Once the turn is made, how long will it be before the ship reaches Barbados if the same 15-knot speed is maintained?



49. Finding the Length of a Guy Wire The height of a radio tower is 500 feet, and the ground on one side of the tower slopes upward at an angle of 10° (see the figure).

(a) How long should a guy wire be if it is to connect to the top of the tower and be secured at a point on the sloped side 100 feet from the base of the tower?

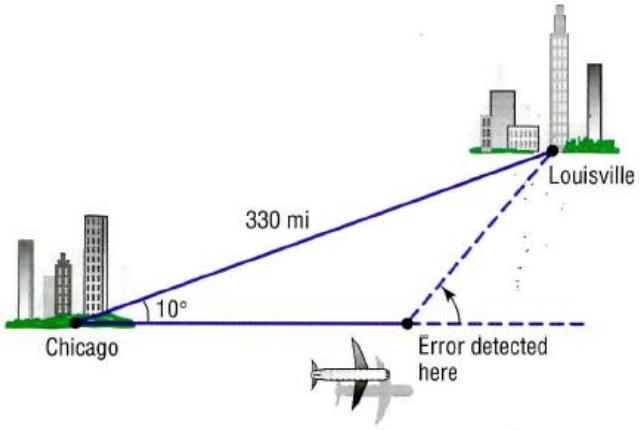
(b) How long should a second guy wire be if it is to connect to the middle of the tower and be secured at a point 100 feet from the base on the flat side?



46. Revising a Flight Plan In attempting to fly from Chicago to Louisville, a distance of 330 miles, a pilot inadvertently took a course that was 10° in error, as indicated in the figure.

(a) If the aircraft maintains an average speed of 220 miles per hour and if the error in direction is discovered after 15 minutes, through what angle should the pilot turn to head toward Louisville?

(b) What new average speed should the pilot maintain so that the total time of the trip is 90 minutes?



47. Major League Baseball Field A Major League baseball diamond is actually a square 90 feet on a side. The pitching rubber is located 60.5 feet from home plate on a line joining home plate and second base.

(a) How far is it from the pitching rubber to first base?

(b) How far is it from the pitching rubber to second base?

(c) If a pitcher faces home plate, through what angle does he need to turn to face first base?

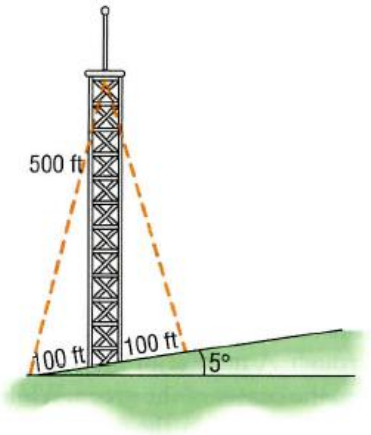
48. Little League Baseball Field According to Little League baseball official regulations, the diamond is a square 60 feet on a side. The pitching rubber is located 46 feet from home plate on a line joining home plate and second base.

(a) How far is it from the pitching rubber to first base?

(b) How far is it from the pitching rubber to second base?

(c) If a pitcher faces home plate, through what angle does he need to turn to face first base?

50. Finding the Length of a Guy Wire A radio tower 500 feet high is located on the side of a hill with an inclination to the horizontal of 5°. See the figure. How long should two guy wires be if they are to connect to the top of the tower and be secured at two points 100 feet directly above and directly below the base of the tower?



51. Wrigley Field, Home of the Chicago Cubs The distance from home plate to the fence in dead center in Wrigley Field is 400 feet (see the figure). How far is it from the fence in dead center to third base?

52. Little League Baseball The distance from home plate to the fence in dead center at the Oak Lawn Little League field is 280 feet. How far is it from the fence in dead center to third base?

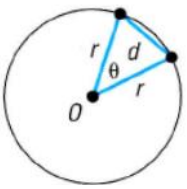
[**Hint:** The distance between the bases in Little League is 60 feet.]

53. Building a Swing Set Clint is building a wooden swing set for his children. Each supporting end of the swing set is to be an A-frame constructed with two 10-foot-long 4-by-4s joined at a 45° angle. To prevent the swing set from tipping over, Clint wants to secure the base of each A-frame to concrete footings. How far apart should the footings for each A-frame be?

55. Geometry Show that the length d of a chord of a circle of radius r is given by the formula

$$d = 2r \sin \frac{\theta}{2}$$

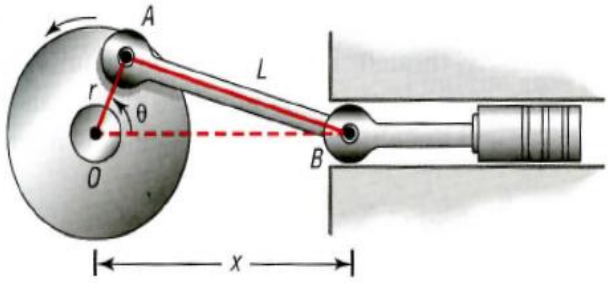
where θ is the central angle formed by the radii to the ends of the chord. See the figure. Use this result to derive the fact that $\sin \theta < \theta$, where $\theta > 0$ is measured in radians.



54. Rods and Pistons Rod OA rotates about the fixed point O so that point A travels on a circle of radius r. Connected to point A is another rod AB of length $L > 2r$, and point B is connected to a piston. See the figure. Show that the distance x between point O and point B is given by

$$x = r\cos\theta + \sqrt{r^2\cos^2\theta + L^2 - r^2}$$

where θ is the angle of rotation of rod OA.



56. For any triangle, show that

$$\cos\frac{C}{2} = \sqrt{\frac{s(s-c)}{ab}}$$

where $s = \frac{1}{2}(a + b + c)$.

[**Hint:** Use a Half-angle Formula and the Law of Cosines.]

57. For any triangle show that

$$\sin\frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{ab}}$$

where $s = \frac{1}{2}(a + b + c)$.

58. Use the Law of Cosines to prove the identity

$$\frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c} = \frac{a^2 + b^2 + c^2}{2abc}$$

Explaining Concepts: Discussion and Writing

59. What do you do first if you are asked to solve a triangle and are given two sides and the included angle?

60. What do you do first if you are asked to solve a triangle and are given three sides?

61. Make up an applied problem that requires using the Law of Cosines.

62. Write down your strategy for solving an oblique triangle.

63. State the Law of Cosines in words.

'Are You Prepared?' Answers

1. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

2. $\theta = 45°$ or $\frac{\pi}{4}$