

7.4 Assess Your Understanding

'Are You Prepared?' Answers are given at the end of these exercises. If you get a wrong answer, read the pages listed in red.

1. **True or False** $\sin^2 \theta = 1 - \cos^2 \theta$. (p. 390)

2. **True or False** $\sin(-\theta) + \cos(-\theta) = \cos \theta - \sin \theta$. (p. 394)

Concepts and Vocabulary

3. Suppose that f and g are two functions with the same domain. If $f(x) = g(x)$ for every x in the domain, the equation is called a(n) _____. Otherwise, it is called a(n) _____ equation.

4. $\tan^2 \theta - \sec^2 \theta =$ _____.

5. $\cos(-\theta) - \cos \theta =$ _____.

6. **True or False** $\sin(-\theta) + \sin \theta = 0$ for any value of θ .

7. **True or False** In establishing an identity, it is often easiest to just multiply both sides by a well-chosen nonzero expression involving the variable.

8. **True or False** $\tan \theta \cdot \cos \theta = \sin \theta$ for any $\theta \neq (2k + 1)\frac{\pi}{2}$.

Skill Building

In Problems 9–18, simplify each trigonometric expression by following the indicated direction.

9. Rewrite in terms of sine and cosine functions:

$$\tan \theta \cdot \csc \theta$$

10. Rewrite in terms of sine and cosine functions:

$$\cot \theta \cdot \sec \theta$$

11. Multiply $\frac{\cos \theta}{1 - \sin \theta}$ by $\frac{1 + \sin \theta}{1 + \sin \theta}$.

12. Multiply $\frac{\sin \theta}{1 + \cos \theta}$ by $\frac{1 - \cos \theta}{1 - \cos \theta}$.

13. Rewrite over a common denominator:

$$\frac{\sin \theta + \cos \theta}{\cos \theta} + \frac{\cos \theta - \sin \theta}{\sin \theta}$$

14. Rewrite over a common denominator:

$$\frac{1}{1 - \cos v} + \frac{1}{1 + \cos v}$$

15. Multiply and simplify: $\frac{(\sin \theta + \cos \theta)(\sin \theta + \cos \theta) - 1}{\sin \theta \cos \theta}$

16. Multiply and simplify: $\frac{(\tan \theta + 1)(\tan \theta + 1) - \sec^2 \theta}{\tan \theta}$

17. Factor and simplify: $\frac{3 \sin^2 \theta + 4 \sin \theta + 1}{\sin^2 \theta + 2 \sin \theta + 1}$

18. Factor and simplify: $\frac{\cos^2 \theta - 1}{\cos^2 \theta - \cos \theta}$

In Problems 19–98, establish each identity.

19. $\csc \theta \cdot \cos \theta = \cot \theta$

22. $1 + \cot^2(-\theta) = \csc^2 \theta$

25. $\tan u \cot u - \cos^2 u = \sin^2 u$

28. $(\csc \theta - 1)(\csc \theta + 1) = \cot^2 \theta$

31. $\cos^2 \theta(1 + \tan^2 \theta) = 1$

34. $\tan^2 \theta \cos^2 \theta + \cot^2 \theta \sin^2 \theta = 1$

37. $\sec u - \tan u = \frac{\cos u}{1 + \sin u}$

40. $9 \sec^2 \theta - 5 \tan^2 \theta = 5 + 4 \sec^2 \theta$

43. $\frac{1 + \tan v}{1 - \tan v} = \frac{\cot v + 1}{\cot v - 1}$

46. $\frac{\csc \theta - 1}{\cot \theta} = \frac{\cot \theta}{\csc \theta + 1}$

49. $\frac{1 - \sin v}{\cos v} + \frac{\cos v}{1 - \sin v} = 2 \sec v$

52. $1 - \frac{\sin^2 \theta}{1 + \cos \theta} = \cos \theta$

20. $\sec \theta \cdot \sin \theta = \tan \theta$

23. $\cos \theta(\tan \theta + \cot \theta) = \csc \theta$

26. $\sin u \csc u - \cos^2 u = \sin^2 u$

29. $(\sec \theta + \tan \theta)(\sec \theta - \tan \theta) = 1$

32. $(1 - \cos^2 \theta)(1 + \cot^2 \theta) = 1$

35. $\sec^4 \theta - \sec^2 \theta = \tan^4 \theta + \tan^2 \theta$

38. $\csc u - \cot u = \frac{\sin u}{1 + \cos u}$

41. $1 - \frac{\cos^2 \theta}{1 + \sin \theta} = \sin \theta$

44. $\frac{\csc v - 1}{\csc v + 1} = \frac{1 - \sin v}{1 + \sin v}$

47. $\frac{1 + \sin \theta}{1 - \sin \theta} = \frac{\csc \theta + 1}{\csc \theta - 1}$

50. $\frac{\cos v}{1 + \sin v} + \frac{1 + \sin v}{\cos v} = 2 \sec v$

53. $\frac{1 - \sin \theta}{1 + \sin \theta} = (\sec \theta - \tan \theta)^2$

21. $1 + \tan^2(-\theta) = \sec^2 \theta$

24. $\sin \theta(\cot \theta + \tan \theta) = \sec \theta$

27. $(\sec \theta - 1)(\sec \theta + 1) = \tan^2 \theta$

30. $(\csc \theta + \cot \theta)(\csc \theta - \cot \theta) = 1$

33. $(\sin \theta + \cos \theta)^2 + (\sin \theta - \cos \theta)^2 = 2$

36. $\csc^4 \theta - \csc^2 \theta = \cot^4 \theta + \cot^2 \theta$

39. $3 \sin^2 \theta + 4 \cos^2 \theta = 3 + \cos^2 \theta$

42. $1 - \frac{\sin^2 \theta}{1 - \cos \theta} = -\cos \theta$

45. $\frac{\sec \theta}{\csc \theta} + \frac{\sin \theta}{\cos \theta} = 2 \tan \theta$

48. $\frac{\cos \theta + 1}{\cos \theta - 1} = \frac{1 + \sec \theta}{1 - \sec \theta}$

51. $\frac{\sin \theta}{\sin \theta - \cos \theta} = \frac{1}{1 - \cot \theta}$

54. $\frac{1 - \cos \theta}{1 + \cos \theta} = (\csc \theta - \cot \theta)^2$

$$55. \frac{\cos \theta}{1 - \tan \theta} + \frac{\sin \theta}{1 - \cot \theta} = \sin \theta + \cos \theta$$

$$56. \frac{\cot \theta}{1 - \tan \theta} + \frac{\tan \theta}{1 - \cot \theta} = 1 + \tan \theta + \cot \theta$$

$$57. \tan \theta + \frac{\cos \theta}{1 + \sin \theta} = \sec \theta$$

$$58. \frac{\sin \theta \cos \theta}{\cos^2 \theta - \sin^2 \theta} = \frac{\tan \theta}{1 - \tan^2 \theta}$$

$$59. \frac{\tan \theta + \sec \theta - 1}{\tan \theta - \sec \theta + 1} = \tan \theta + \sec \theta$$

$$60. \frac{\sin \theta - \cos \theta + 1}{\sin \theta + \cos \theta - 1} = \frac{\sin \theta + 1}{\cos \theta}$$

$$61. \frac{\tan \theta - \cot \theta}{\tan \theta + \cot \theta} = \sin^2 \theta - \cos^2 \theta$$

$$62. \frac{\sec \theta - \cos \theta}{\sec \theta + \cos \theta} = \frac{\sin^2 \theta}{1 + \cos^2 \theta}$$

$$63. \frac{\tan u - \cot u}{\tan u + \cot u} + 1 = 2 \sin^2 u$$

$$64. \frac{\tan u - \cot u}{\tan u + \cot u} + 2 \cos^2 u = 1$$

$$65. \frac{\sec \theta + \tan \theta}{\cot \theta + \cos \theta} = \tan \theta \sec \theta$$

$$66. \frac{\sec \theta}{1 + \sec \theta} = \frac{1 - \cos \theta}{\sin^2 \theta}$$

$$67. \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} + 1 = 2 \cos^2 \theta$$

$$68. \frac{1 - \cot^2 \theta}{1 + \cot^2 \theta} + 2 \cos^2 \theta = 1$$

$$69. \frac{\sec \theta - \csc \theta}{\sec \theta \csc \theta} = \sin \theta - \cos \theta$$

$$70. \frac{\sin^2 \theta - \tan \theta}{\cos^2 \theta - \cot \theta} = \tan^2 \theta$$

$$71. \sec \theta - \cos \theta = \sin \theta \tan \theta$$

$$72. \tan \theta + \cot \theta = \sec \theta \csc \theta$$

$$73. \frac{1}{1 - \sin \theta} + \frac{1}{1 + \sin \theta} = 2 \sec^2 \theta$$

$$74. \frac{1 + \sin \theta}{1 - \sin \theta} - \frac{1 - \sin \theta}{1 + \sin \theta} = 4 \tan \theta \sec \theta$$

$$75. \frac{\sec \theta}{1 - \sin \theta} = \frac{1 + \sin \theta}{\cos^3 \theta}$$

$$76. \frac{1 + \sin \theta}{1 - \sin \theta} = (\sec \theta + \tan \theta)^2$$

$$77. \frac{(\sec v - \tan v)^2 + 1}{\csc v (\sec v - \tan v)} = 2 \tan v$$

$$78. \frac{\sec^2 v - \tan^2 v + \tan v}{\sec v} = \sin v + \cos v$$

$$79. \frac{\sin \theta + \cos \theta}{\cos \theta} - \frac{\sin \theta - \cos \theta}{\sin \theta} = \sec \theta \csc \theta$$

$$80. \frac{\sin \theta + \cos \theta}{\sin \theta} - \frac{\cos \theta - \sin \theta}{\cos \theta} = \sec \theta \csc \theta$$

$$81. \frac{\sin^3 \theta + \cos^3 \theta}{\sin \theta + \cos \theta} = 1 - \sin \theta \cos \theta$$

$$82. \frac{\sin^3 \theta + \cos^3 \theta}{1 - 2 \cos^2 \theta} = \frac{\sec \theta - \sin \theta}{\tan \theta - 1}$$

$$83. \frac{\cos^2 \theta - \sin^2 \theta}{1 - \tan^2 \theta} = \cos^2 \theta$$

$$84. \frac{\cos \theta + \sin \theta - \sin^3 \theta}{\sin \theta} = \cot \theta + \cos^2 \theta$$

$$85. \frac{(2 \cos^2 \theta - 1)^2}{\cos^4 \theta - \sin^4 \theta} = 1 - 2 \sin^2 \theta$$

$$86. \frac{1 - 2 \cos^2 \theta}{\sin \theta \cos \theta} = \tan \theta - \cot \theta$$

$$87. \frac{1 + \sin \theta + \cos \theta}{1 + \sin \theta - \cos \theta} = \frac{1 + \cos \theta}{\sin \theta}$$

$$88. \frac{1 + \cos \theta + \sin \theta}{1 + \cos \theta - \sin \theta} = \sec \theta + \tan \theta$$

$$89. (a \sin \theta + b \cos \theta)^2 + (a \cos \theta - b \sin \theta)^2 = a^2 + b^2$$

$$90. (2a \sin \theta \cos \theta)^2 + a^2 (\cos^2 \theta - \sin^2 \theta)^2 = a^2$$

$$91. \frac{\tan \alpha + \tan \beta}{\cot \alpha + \cot \beta} = \tan \alpha \tan \beta$$

$$92. (\tan \alpha + \tan \beta)(1 - \cot \alpha \cot \beta) + (\cot \alpha + \cot \beta)(1 - \tan \alpha \tan \beta) = 0$$

$$93. (\sin \alpha + \cos \beta)^2 + (\cos \beta + \sin \alpha)(\cos \beta - \sin \alpha) = 2 \cos \beta (\sin \alpha + \cos \beta)$$

$$94. (\sin \alpha - \cos \beta)^2 + (\cos \beta + \sin \alpha)(\cos \beta - \sin \alpha) = -2 \cos \beta (\sin \alpha - \cos \beta)$$

$$95. \ln |\sec \theta| = -\ln |\cos \theta|$$

$$96. \ln |\tan \theta| = \ln |\sin \theta| - \ln |\cos \theta|$$

$$97. \ln |1 + \cos \theta| + \ln |1 - \cos \theta| = 2 \ln |\sin \theta|$$

$$98. \ln |\sec \theta + \tan \theta| + \ln |\sec \theta - \tan \theta| = 0$$

In Problems 99–102, show that the functions f and g are identically equal.

99. $f(x) = \sin x \cdot \tan x$ $g(x) = \sec x - \cos x$

100. $f(x) = \cos x \cdot \cot x$ $g(x) = \csc x - \sin x$

101. $f(\theta) = \frac{1 - \sin \theta}{\cos \theta} - \frac{\cos \theta}{1 + \sin \theta}$ $g(\theta) = 0$

102. $f(\theta) = \tan \theta + \sec \theta$ $g(\theta) = \frac{\cos \theta}{1 - \sin \theta}$

103. Show $\sqrt{16 + 16 \tan^2 \theta} = 4 \sec \theta$ if $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

104. Show $\sqrt{9 \sec^2 \theta - 9} = 3 \tan \theta$ if $\pi \leq \theta < \frac{3\pi}{2}$.

Applications and Extensions

105. Searchlights A searchlight at the grand opening of a new car dealership casts a spot of light on a wall located 75 meters from the searchlight. The acceleration \ddot{r} of the spot of light is found to be $\ddot{r} = 1200 \sec \theta (2 \sec^2 \theta - 1)$. Show that this is equivalent to $\ddot{r} = 1200 \left(\frac{1 + \sin^2 \theta}{\cos^3 \theta} \right)$.

106. Optical Measurement Optical methods of measurement often rely on the interference of two light waves. If two light waves, identical except for a phase lag, are mixed together, the resulting intensity, or irradiance, is given by $I_t = 4A^2 \frac{(\csc \theta - 1)(\sec \theta + \tan \theta)}{\csc \theta \sec \theta}$. Show that this is equivalent to $I_t = (2A \cos \theta)^2$.

Source: Adapted from Hibbeler, *Engineering Mechanics: Dynamics*, 10th ed., Prentice Hall © 2004.

Source: *Experimental Techniques*, July/August 2002

Explaining Concepts: Discussion and Writing

107. Write a few paragraphs outlining your strategy for establishing identities.

109. Why do you think it is usually preferable to start with the side containing the more complicated expression when establishing an identity?

108. Write down the three Pythagorean Identities.

110. Make up an identity that is not a Fundamental Identity.

'Are You Prepared?' Answers

1. True 2. True