## 4.4 Assess Your Understanding

### 'Are You Prepared?'

*Answers are given at the end of these exercises. If you get a wrong answer, read the pages listed in red.*

1. ***True or False*** The quotient of two polynomial expressions is a rational expression. (pp. A35)
2. What are the quotient and remainder when $3x^4 - x^2$ is divided by $x^3 - x^2 + 1$? (pp. A24–A26)
3. Graph $y = \frac{1}{x}$. (pp. 22–23)
4. Graph $y = 2(x + 1)^2 - 3$ using transformations. (pp. 104–108)

### Concepts and Vocabulary

5. ***True or False*** The domain of every rational function is the set of all real numbers.
6. If, as $x \to -\infty$ or as $x \to \infty$, the values of $R(x)$ approach some fixed number $L$, then the line $y = L$ is a ________ ________ of the graph of $R$.
7. If, as $x$ approaches some number $c$, the values of $|R(x)| \to \infty$, then the line $x = c$ is a ________ ________ of the graph of $R$.
8. For a rational function $R$, if the degree of the numerator is less than the degree of the denominator, then $R$ is ________.
9. ***True or False*** The graph of a rational function may intersect a horizontal asymptote.
10. ***True or False*** The graph of a rational function may intersect a vertical asymptote.
11. If a rational function is proper, then ______ is a horizontal asymptote.
12. ***True or False*** If the degree of the numerator of a rational function equals the degree of the denominator, then the ratio of the leading coefficients gives rise to the horizontal asymptote.

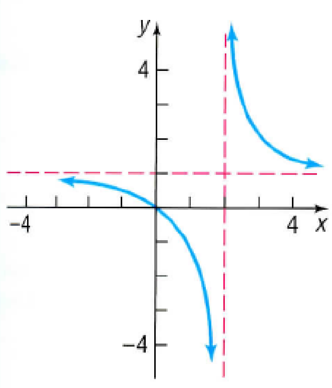
### Skill Building

*In Problems 13–24, find the domain of each rational function.*

13. $R(x) = \dfrac{4x}{x - 3}$
14. $R(x) = \dfrac{5x^2}{3 + x}$
15. $H(x) = \dfrac{-4x^2}{(x - 2)(x + 4)}$
16. $G(x) = \dfrac{6}{(x + 3)(4 - x)}$
17. $F(x) = \dfrac{3x(x - 1)}{2x^2 - 5x - 3}$
18. $Q(x) = \dfrac{-x(1 - x)}{3x^2 + 5x - 2}$
19. $R(x) = \dfrac{x}{x^3 - 8}$
20. $R(x) = \dfrac{x}{x^4 - 1}$
21. $H(x) = \dfrac{3x^2 + x}{x^2 + 4}$
22. $G(x) = \dfrac{x - 3}{x^4 + 1}$
23. $R(x) = \dfrac{3(x^2 - x - 6)}{4(x^2 - 9)}$
24. $F(x) = \dfrac{-2(x^2 - 4)}{3(x^2 + 4x + 4)}$

*In Problems 25–30, use the graph shown to find*
*(a) The domain and range of each function*
*(b) The intercepts, if any*
*(c) Horizontal asymptote, if one exists*
*(d) Vertical asymptotes, if any*
*(e) Oblique asymptote, if one exists*

25.

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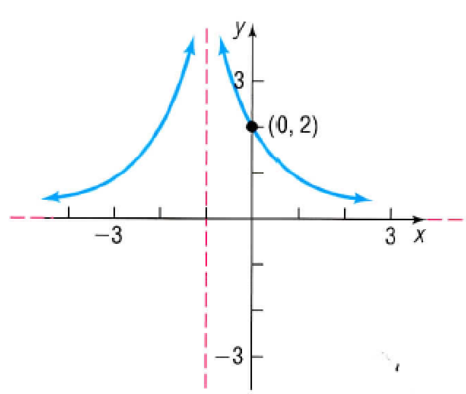


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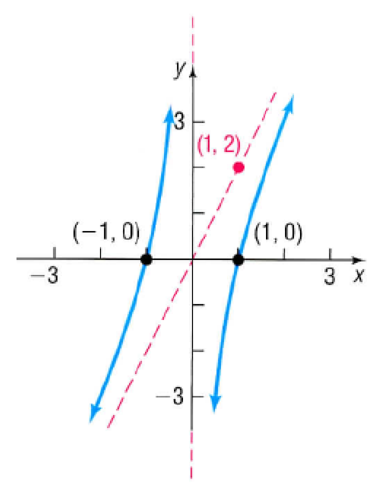

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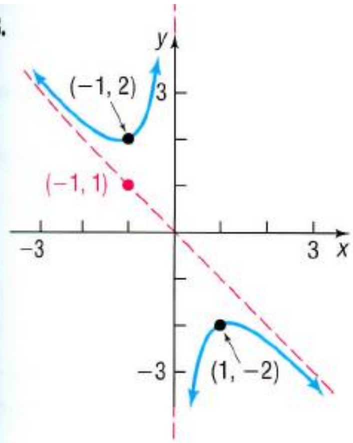


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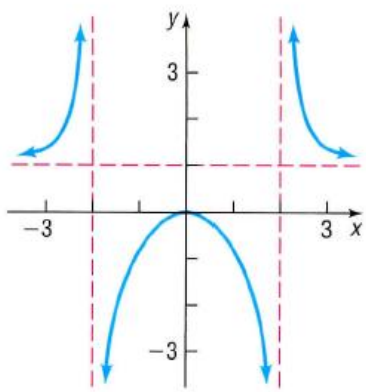


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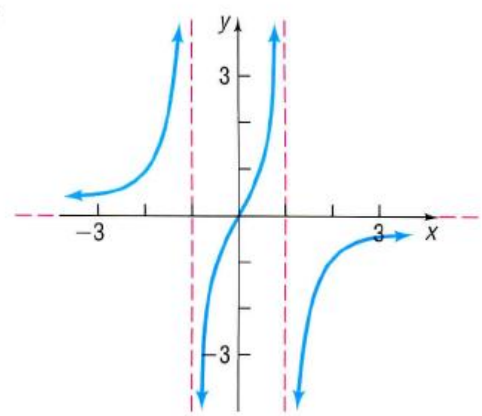



*In Problems 31–42, graph each rational function using transformations.*

31. $F(x) = 2 + \frac{1}{x}$

32. $Q(x) = 3 + \frac{1}{x^2}$

33. $R(x) = \frac{1}{(x-1)^2}$

34. $R(x) = \frac{3}{x}$

35. $H(x) = \frac{-2}{x+1}$

36. $G(x) = \frac{2}{(x+2)^2}$

37. $R(x) = \frac{-1}{x^2 + 4x + 4}$

38. $R(x) = \frac{1}{x-1} + 1$

39. $G(x) = 1 + \frac{2}{(x-3)^2}$

40. $F(x) = 2 - \frac{1}{x+1}$

41. $R(x) = \frac{x^2 - 4}{x^2}$

42. $R(x) = \frac{x-4}{x}$

*In Problems 43–54, find the vertical, horizontal, and oblique asymptotes, if any, of each rational function.*

43. $R(x) = \frac{3x}{x+4}$

44. $R(x) = \frac{3x+5}{x-6}$

45. $H(x) = \frac{x^3 - 8}{x^2 - 5x + 6}$

46. $G(x) = \frac{x^3 + 1}{x^2 - 5x - 14}$

47. $T(x) = \frac{x^3}{x^4 - 1}$

48. $P(x) = \frac{4x^2}{x^3 - 1}$

49. $Q(x) = \frac{2x^2 - 5x - 12}{3x^2 - 11x - 4}$

50. $F(x) = \frac{x^2 + 6x + 5}{2x^2 + 7x + 5}$

51. $R(x) = \frac{6x^2 + 7x - 5}{3x + 5}$

52. $R(x) = \frac{8x^2 + 26x - 7}{4x - 1}$

53. $G(x) = \frac{x^4 - 1}{x^2 - x}$

54. $F(x) = \frac{x^4 - 16}{x^2 - 2x}$

## Applications and Extensions

55. **Gravity** In physics, it is established that the acceleration due to gravity, $g$ (in meters/sec$^2$), at a height $h$ meters above sea level is given by

$$g(h) = \frac{3.99 \times 10^{14}}{(6.374 \times 10^6 + h)^2}$$

where $6.374 \times 10^6$ is the radius of Earth in meters.

(a) What is the acceleration due to gravity at sea level?

(b) The Willis Tower in Chicago, Illinois, is 443 meters tall. What is the acceleration due to gravity at the top of the Willis Tower?

(c) The peak of Mount Everest is 8848 meters above sea level. What is the acceleration due to gravity on the peak of Mount Everest?

(d) Find the horizontal asymptote of $g(h)$.

(e) Solve $g(h) = 0$. How do you interpret your answer?

56. **Population Model** A rare species of insect was discovered in the Amazon Rain Forest. To protect the species, environmentalists declared the insect endangered and transplanted the insect into a protected area. The population $P$ of the insect $t$ months after being transplanted is

(a) Let $R_1 = 10$ ohms, and graph $R_{tot}$ as a function of $R_2$.

(b) Find and interpret any asymptotes of the graph obtained in part (a).

(c) If $R_2 = 2\sqrt{R_1}$, what value of $R_1$ will yield an $R_{tot}$ of 17 ohms?

***Source:*** *en.wikipedia.org/wiki/Series_and_parallel_circuits*

58. **Newton's Method** In calculus you will learn that, if

$$p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$$

is a polynomial function, then the *derivative* of $p(x)$ is

$$p'(x) = na_n x^{n-1} + (n-1)a_{n-1}x^{n-2} + \cdots + 2a_2 x + a_1$$

**Newton's Method** is an efficient method for approximating the $x$-intercepts (or real zeros) of a function, such as $p(x)$. The following steps outline Newton's Method.

**Step 1:** Select an initial value $x_0$ that is somewhat close to the $x$-intercept being sought.

**Step 2:** Find values for $x$ using the relation

$$x_{n+1} = x_n - \frac{p(x_n)}{p'(x_n)} \quad n = 1, 2, \ldots$$

$$P(t) = \frac{50(1 + 0.5t)}{2 + 0.01t}$$

(a) How many insects were discovered? In other words, what was the population when $t = 0$?
(b) What will the population be after 5 years?
(c) Determine the horizontal asymptote of $P(t)$. What is the largest population that the protected area can sustain?

**57. Resistance in Parallel Circuits** From Ohm's law for circuits, it follows that the total resistance $R_{tot}$ of two components connected in parallel is given by the equation

$$R_{tot} = \frac{R_1 R_2}{R_1 + R_2}$$

where $R_1$ and $R_2$ are the individual resistances.

until you get two consecutive values $x_n$ and $x_{n+1}$ that agree to whatever decimal place accuracy you desire.

**STEP 3:** The approximate zero will be $x_{n+1}$.

Consider the polynomial function $p(x) = x^3 - 7x - 40$.
(a) Evaluate $p(5)$ and $p(-3)$.
(b) What might we conclude about a zero of $p$? Explain.
(c) Use Newton's Method to approximate an $x$-intercept, $r$, $-3 < r < 5$, of $p(x)$ to four decimal places.
(d) Use a graphing utility to graph $p(x)$ and verify your answer in part (c).
(e) Using a graphing utility, evaluate $p(r)$ to verify your result.

## Explaining Concepts: Discussion and Writing

**59.** If the graph of a rational function $R$ has the vertical asymptote $x = 4$, the factor $x - 4$ must be present in the denominator of $R$. Explain why.

**60.** If the graph of a rational function $R$ has the horizontal asymptote $y = 2$, the degree of the numerator of $R$ equals the degree of the denominator of $R$. Explain why.

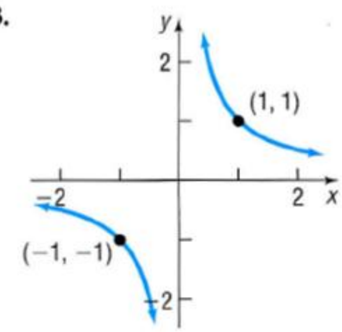
**61.** Can the graph of a rational function have both a horizontal and an oblique asymptote? Explain.

**62.** Make up a rational function that has $y = 2x + 1$ as an oblique asymptote. Explain the methodology that you used.

## 'Are You Prepared?' Answers

**1.** True **2.** Quotient: $3x + 3$; remainder: $2x^2 - 3x - 3$

**3.**




**4.**