4.3 Assess Your Understanding

'Are You Prepared?' *Answers are given at the end of these exercises. If you get a wrong answer, read the pages listed in red.*

1. Find the sum and the product of the complex numbers $3 - 2i$ and $-3 + 5i$. (pp. A56–A57)
2. In the complex number system, find the complex solutions of the equation $x^2 + 2x + 2 = 0$. (pp. A60–A61)

Concepts and Vocabulary

3. Every polynomial function of odd degree with real coefficients has at least _____ real zero(s).
4. If $3 + 4i$ is a zero of a polynomial function of degree 5 with real coefficients, then so is _____.
5. ***True or False*** A polynomial function of degree n with real coefficients has exactly n complex zeros. At most n of them are real zeros.
6. ***True or False*** A polynomial function of degree 4 with real coefficients could have $-3, 2 + i, 2 - i$, and $-3 + 5i$ as its zeros.

Skill Building

In Problems 7–16, information is given about a polynomial function $f(x)$ whose coefficients are real numbers. Find the remaining zeros of f.

7. Degree 3; zeros: $3, 4 - i$
8. Degree 3; zeros: $4, 3 + i$
9. Degree 4; zeros: $i, 1 + i$
10. Degree 4; zeros: $1, 2, 2 + i$
11. Degree 5; zeros: $1, i, 2i$
12. Degree 5; zeros: $0, 1, 2, i$
13. Degree 4; zeros: $i, 2, -2$
14. Degree 4; zeros: $2 - i, -i$
15. Degree 6; zeros: $2, 2 + i, -3 - i, 0$
16. Degree 6; zeros: $i, 3 - 2i, -2 + i$

In Problems 17–22, form a polynomial function $f(x)$ with real coefficients having the given degree and zeros. Answers will vary depending on the choice of the leading coefficient. Use a graphing utility to graph the function and verify the result.

17. Degree 4; zeros: $3 + 2i$; 4, multiplicity 2
18. Degree 4; zeros: $i; 1 + 2i$
19. Degree 5; zeros: $2; -i; 1 + i$
20. Degree 6; zeros: $i; 4 - i; 2 + i$
21. Degree 4; zeros: 3, multiplicity 2; $-i$
22. Degree 5; zeros: 1, multiplicity 3; $1 + i$

In Problems 23–30, use the given zero to find the remaining zeros of each function.

23. $f(x) = x^3 - 4x^2 + 4x - 16$; zero: $2i$
24. $g(x) = x^3 + 3x^2 + 25x + 75$; zero: $-5i$
25. $f(x) = 2x^4 + 5x^3 + 5x^2 + 20x - 12$; zero: $-2i$
26. $h(x) = 3x^4 + 5x^3 + 25x^2 + 45x - 18$; zero: $3i$
27. $h(x) = x^4 - 9x^3 + 21x^2 + 21x - 130$; zero: $3 - 2i$
28. $f(x) = x^4 - 7x^3 + 14x^2 - 38x - 60$; zero: $1 + 3i$
29. $h(x) = 3x^5 + 2x^4 + 15x^3 + 10x^2 - 528x - 352$; zero: $-4i$
30. $g(x) = 2x^5 - 3x^4 - 5x^3 - 15x^2 - 207x + 108$; zero: $3i$

In Problems 31–40, find the complex zeros of each polynomial function. Write f in factored form.

31. $f(x) = x^3 - 1$
32. $f(x) = x^4 - 1$
33. $f(x) = x^3 - 8x^2 + 25x - 26$
34. $f(x) = x^3 + 13x^2 + 57x + 85$
35. $f(x) = x^4 + 5x^2 + 4$
36. $f(x) = x^4 + 13x^2 + 36$
37. $f(x) = x^4 + 2x^3 + 22x^2 + 50x - 75$
38. $f(x) = x^4 + 3x^3 - 19x^2 + 27x - 252$
39. $f(x) = 3x^4 - x^3 - 9x^2 + 159x - 52$
40. $f(x) = 2x^4 + x^3 - 35x^2 - 113x + 65$

Explaining Concepts: Discussion and Writing

In Problems 41 and 42, explain why the facts given are contradictory.

41. $f(x)$ is a polynomial function of degree 3 whose coefficients are real numbers; its zeros are $4 + i, 4 - i$, and $2 + i$.
42. $f(x)$ is a polynomial function of degree 3 whose coefficients are real numbers; its zeros are $2, i$, and $3 + i$.
43. $f(x)$ is a polynomial function of degree 4 whose coefficients are real numbers; three of its zeros are $2, 1 + 2i$, and $1 - 2i$. Explain why the remaining zero must be a real number.
44. $f(x)$ is a polynomial function of degree 4 whose coefficients are real numbers; two of its zeros are -3 and $4 - i$. Explain why one of the remaining zeros must be a real number. Write down one of the missing zeros.

'Are You Prepared?' Answers

1. Sum: $3i$; product: $1 + 21i$
2. $-1 - i, -1 + i$