4.2 Assess Your Understanding

'Are You Prepared?' *Answers are given at the end of these exercises. If you get a wrong answer, read the pages listed in red.*

1. Find $f(-1)$ if $f(x) = 2x^2 - x$. (pp. 62–63)
2. Factor the expression $6x^2 + x - 2$. (pp. A27–A28)
3. Find the quotient and remainder if $3x^4 - 5x^3 + 7x - 4$ is divided by $x - 3$. (pp. A24–A26 or A31–A33)
4. Find the zeros of $f(x) = x^2 + x - 3$. (pp. 150–152)

Concepts and Vocabulary

5. In the process of polynomial division, (Divisor)(Quotient) + _____ = _____.
6. When a polynomial function f is divided by $x - c$, the remainder is _____.
7. If a function f, whose domain is all real numbers, is even and if 4 is a zero of f, then _____ is also a zero.
8. ***True or False*** Every polynomial function of degree 3 with real coefficients has exactly three real zeros.
9. If f is a polynomial function and $x - 4$ is a factor of f, then $f(4) =$ _____.
10. ***True or False*** If f is a polynomial function of degree 4 and if $f(2) = 5$, then

$$\frac{f(x)}{x-2} = p(x) + \frac{5}{x-2}$$

where $p(x)$ is a polynomial function of degree 3.

Skill Building

In Problems 11–20, use the Remainder Theorem to find the remainder when $f(x)$ is divided by $x - c$. Then use the Factor Theorem to determine whether $x - c$ is a factor of $f(x)$.

11. $f(x) = 4x^3 - 3x^2 - 8x + 4; x - 2$
12. $f(x) = -4x^3 + 5x^2 + 8; \quad x + 3$
13. $f(x) = 3x^4 - 6x^3 - 5x + 10; x - 2$
14. $f(x) = 4x^4 - 15x^2 - 4; x - 2$
15. $f(x) = 3x^6 + 82x^3 + 27; x + 3$
16. $f(x) = 2x^6 - 18x^4 + x^2 - 9; x + 3$
17. $f(x) = 4x^6 - 64x^4 + x^2 - 15; x + 4$
18. $f(x) = x^6 - 16x^4 + x^2 - 16; x + 4$
19. $f(x) = 2x^4 - x^3 + 2x - 1; x - \frac{1}{2}$
20. $f(x) = 3x^4 + x^3 - 3x + 1; x + \frac{1}{3}$

In Problems 21–32, determine the maximum number of real zeros that each polynomial function may have. Then list the potential rational zeros of each polynomial function. Do not attempt to find the zeros.

21. $f(x) = 3x^4 - 3x^3 + x^2 - x + 1$
22. $f(x) = x^5 - x^4 + 2x^2 + 3$
23. $f(x) = x^5 - 6x^2 + 9x - 3$
24. $f(x) = 2x^5 - x^4 - x^2 + 1$
25. $f(x) = -4x^3 - x^2 + x + 2$
26. $f(x) = 6x^4 - x^2 + 2$
27. $f(x) = 6x^4 - x^2 + 9$
28. $f(x) = -4x^3 + x^2 + x + 6$
29. $f(x) = 2x^5 - x^3 + 2x^2 + 12$
30. $f(x) = 3x^5 - x^2 + 2x + 18$
31. $f(x) = 6x^4 + 2x^3 - x^2 + 20$
32. $f(x) = -6x^3 - x^2 + x + 10$

In Problems 33–38, find the bounds to the zeros of each polynomial function. Use the bounds to obtain a complete graph of f.

33. $f(x) = 2x^3 + x^2 - 1$
34. $f(x) = 3x^3 - 2x^2 + x + 4$
35. $f(x) = x^3 - 5x^2 - 11x + 11$
36. $f(x) = 2x^3 - x^2 - 11x - 6$
37. $f(x) = x^4 + 3x^3 - 5x^2 + 9$
38. $f(x) = 4x^4 - 12x^3 + 27x^2 - 54x + 81$

In Problems 39–56, find the real zeros of f. Use the real zeros to factor f.

39. $f(x) = x^3 + 2x^2 - 5x - 6$
40. $f(x) = x^3 + 8x^2 + 11x - 20$
41. $f(x) = 2x^3 - 13x^2 + 24x - 9$
42. $f(x) = 2x^3 - 5x^2 - 4x + 12$
43. $f(x) = 3x^3 + 4x^2 + 4x + 1$
44. $f(x) = 3x^3 - 7x^2 + 12x - 28$
45. $f(x) = x^3 - 8x^2 + 17x - 6$
46. $f(x) = x^3 + 6x^2 + 6x - 4$
47. $f(x) = x^4 + x^3 - 3x^2 - x + 2$
48. $f(x) = x^4 - x^3 - 6x^2 + 4x + 8$
49. $f(x) = 2x^4 + 17x^3 + 35x^2 - 9x - 45$
50. $f(x) = 4x^4 - 15x^3 - 8x^2 + 15x + 4$
51. $f(x) = 2x^4 - 3x^3 - 21x^2 - 2x + 24$
52. $f(x) = 2x^4 + 11x^3 - 5x^2 - 43x + 35$
53. $f(x) = 4x^4 + 7x^2 - 2$
54. $f(x) = 4x^4 + 15x^2 - 4$
55. $f(x) = 4x^5 - 8x^4 - x + 2$
56. $f(x) = 4x^5 + 12x^4 - x - 3$

In Problems 57–62, find the real zeros of f. If necessary, round to two decimal places.

57. $f(x) = x^3 + 3.2x^2 - 16.83x - 5.31$

58. $f(x) = x^3 + 3.2x^2 - 7.25x - 6.3$

59. $f(x) = x^4 - 1.4x^3 - 33.71x^2 + 23.94x + 292.41$

60. $f(x) = x^4 + 1.2x^3 - 7.46x^2 - 4.692x + 15.2881$

61. $f(x) = x^3 + 19.5x^2 - 1021x + 1000.5$

62. $f(x) = x^3 + 42.2x^2 - 664.8x + 1490.4$

In Problems 63–72, find the real solutions of each equation.

63. $x^4 - x^3 + 2x^2 - 4x - 8 = 0$

64. $2x^3 + 3x^2 + 2x + 3 = 0$

65. $3x^3 + 4x^2 - 7x + 2 = 0$

66. $2x^3 - 3x^2 - 3x - 5 = 0$

67. $3x^3 - x^2 - 15x + 5 = 0$

68. $2x^3 - 11x^2 + 10x + 8 = 0$

69. $x^4 + 4x^3 + 2x^2 - x + 6 = 0$

70. $x^4 - 2x^3 + 10x^2 - 18x + 9 = 0$

71. $x^3 - \frac{2}{3}x^2 + \frac{8}{3}x + 1 = 0$

72. $x^3 - \frac{2}{3}x^2 + 3x - 2 = 0$

In Problems 73–78, use the Intermediate Value Theorem to show that each function has a zero in the given interval. Approximate the zero rounded to two decimal places.

73. $f(x) = 8x^4 - 2x^2 + 5x - 1; \quad [0, 1]$

74. $f(x) = x^4 + 8x^3 - x^2 + 2; \quad [-1, 0]$

75. $f(x) = 2x^3 + 6x^2 - 8x + 2; \quad [-5, -4]$

76. $f(x) = 3x^3 - 10x + 9; \quad [-3, -2]$

77. $f(x) = x^5 - x^4 + 7x^3 - 7x^2 - 18x + 18; \quad [1.4, 1.5]$

78. $f(x) = x^5 - 3x^4 - 2x^3 + 6x^2 + x + 2; \quad [1.7, 1.8]$

Mixed Practice

In Problems 79–86, analyze each polynomial function using Steps 1 through 8 on page 190 in Section 4.1.

79. $f(x) = x^3 + 2x^2 - 5x - 6$
[**Hint:** See Problem 39.]

80. $f(x) = x^3 + 8x^2 + 11x - 20$
[**Hint:** See Problem 40.]

81. $f(x) = x^4 + x^3 - 3x^2 - x + 2$
[**Hint:** See Problem 47.]

82. $f(x) = x^4 - x^3 - 6x^2 + 4x + 8$
[**Hint:** See Problem 48.]

83. $f(x) = 4x^5 - 8x^4 - x + 2$
[**Hint:** See Problem 55.]

84. $f(x) = 4x^5 + 12x^4 - x - 3$
[**Hint:** See Problem 56.]

85. $f(x) = 6x^4 - 37x^3 + 58x^2 + 3x - 18$

86. $f(x) = 20x^4 + 73x^3 + 46x^2 - 52x - 24$

Applications and Extensions

87. Find k such that $f(x) = x^3 - kx^2 + kx + 2$ has the factor $x - 2$.

88. Find k such that $f(x) = x^4 - kx^3 + kx^2 + 1$ has the factor $x + 2$.

89. What is the remainder when $f(x) = 2x^{20} - 8x^{10} + x - 2$ is divided by $x - 1$?

90. What is the remainder when $f(x) = -3x^{17} + x^9 - x^5 + 2x$ is divided by $x + 1$?

91. Use the Factor Theorem to prove that $x - c$ is a factor of $x^n - c^n$ for any positive integer n.

92. Use the Factor Theorem to prove that $x + c$ is a factor of $x^n + c^n$ if $n \geq 1$ is an odd integer.

93. One solution of the equation $x^3 - 8x^2 + 16x - 3 = 0$ is 3. Find the sum of the remaining solutions.

94. One solution of the equation $x^3 + 5x^2 + 5x - 2 = 0$ is -2. Find the sum of the remaining solutions.

95. Geometry What is the length of the edge of a cube if, after a slice 1 inch thick is cut from one side, the volume remaining is 294 cubic inches?

96. Geometry What is the length of the edge of a cube if its volume could be doubled by an increase of 6 centimeters in one edge, an increase of 12 centimeters in a second edge, and a decrease of 4 centimeters in the third edge?

97. Let $f(x)$ be a polynomial function whose coefficients are integers. Suppose that r is a real zero of f and that the leading coefficient of f is 1. Use the Rational Zeros Theorem to show that r is either an integer or an irrational number.

98. Prove the Rational Zeros Theorem.

[**Hint:** Let $\frac{p}{q}$, where p and q have no common factors except 1 and -1, be a zero of the polynomial function

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$$

whose coefficients are all integers. Show that

$$a_n p^n + a_{n-1}p^{n-1}q + \cdots + a_1 pq^{n-1} + a_0 q^n = 0$$

Now, because p is a factor of the first n terms of this equation, p must also be a factor of the term $a_0 q^n$. Since p is not a factor of q (why?), p must be a factor of a_0. Similarly, q must be a factor of a_n.]

99. Bisection Method for Approximating Zeros of a Function f We begin with two consecutive integers, a and $a + 1$, such that $f(a)$ and $f(a + 1)$ are of opposite sign. Evaluate f at the midpoint m_1 of a and $a + 1$. If $f(m_1) = 0$, then m_1 is the zero of f, and we are finished. Otherwise, $f(m_1)$ is of opposite sign to either $f(a)$ or $f(a + 1)$. Suppose that it

is $f(a)$ and $f(m_1)$ that are of opposite sign. Now evaluate f at the midpoint m_2 of a and m_1. Repeat this process until the desired degree of accuracy is obtained. Note that each iteration places the zero in an interval whose length is half that of the previous interval. Use the bisection method to approximate the zero of $f(x) = 8x^4 - 2x^2 + 5x - 1$ in the interval $[0, 1]$ correct to three decimal places. Verify your result using a graphing utility.

[**Hint:** The process ends when both endpoints agree to the desired number of decimal places.]

Discussion and Writing

100. Is $\frac{1}{3}$ a zero of $f(x) = 2x^3 + 3x^2 - 6x + 7$? Explain.

101. Is $\frac{1}{3}$ a zero of $f(x) = 4x^3 - 5x^2 - 3x + 1$? Explain.

102. Is $\frac{3}{5}$ a zero of $f(x) = 2x^6 - 5x^4 + x^3 - x + 1$? Explain.

103. Is $\frac{2}{3}$ a zero of $f(x) = x^7 + 6x^5 - x^4 + x + 2$? Explain.

'Are You Prepared' Answers

1. 3 **2.** $(3x + 2)(2x - 1)$ **3.** Quotient: $3x^3 + 4x^2 + 12x + 43$; remainder: 125 **4.** $\left\{\frac{-1 - \sqrt{13}}{2}, \frac{-1 + \sqrt{13}}{2}\right\}$